The Consequential Implications of Endogenous Schumpeterian Growth for Business Cycles

Work in progress. Comments welcome.

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Abstract

Commonly used DSGE business cycle models focus on the cyclical fluctuations around a constant exogenous trend of output. In this paper, we embed Schumpeterian innovation into the intermediate production sector within a New Keynesian model with nominal wage and price stickiness and we show that endogenous decisions to invest in $R\&D$ have implications for the likelihood of innovating and pushing the technological frontier, while adding a relevant transmission channel for common shocks that are consequential for business cycles. This raises important theoretical, methodological, and empirical points, with both qualitative and quantitative dimensions.

Theoretically, new challenges at the modelling and simulation stages need to be addressed, in particular with respect to the interaction between Schumpeterian innovation and price rigidities, as well as between business cycle and growth. The introduction of Schumpeterian innovation, as the determinant of the expected growth rate of the economy, makes it also conceivable to consider the implications of knowledge-spillover shocks, as an additional dimension not found in usual business cycle models.

Methodologically, the cyclical behaviour of the technological frontier's growth rate points out the importance of explicitly applying the same treatment to both simulated and observed macroeconomic variables. This has ramifications for the interpretation of the cyclical impulse response functions, as well as for the cumulative impact on the levels of macroeconomic variables.

We find that a reasonable calibration of the model makes it possible to get key moments and comovements for important macroeconomic variables that are consistent with their observed counterparts. It reveals also the cyclical impacts of various common shocks as well the knowledge-spillover shocks on macroeconomic variables. In particular, the variables' dynamics is not invariant to the parameter calibration of steady-state endogenous growth. Moreover, there are important welfare implications of different combinations of steady-state innovation probability and extent of knowledge-spillover, for the same steady-state growth rate of the economy. For instance, accounting for dynamic interactions, a 23% quarterly probability of innovating consistent with an annual 1.95% of the technological frontier raises welfare in consumption-equivalent terms by 4.7% higher level, by comparison to a 15% quarterly innovation probability.

KEY WORDS: Schumpeterian endogenous growth; Business cycles; New Keynesian dynamic stochastic general equilibrium (DSGE) model.

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Contents

1 Introduction

Investing in research and development ($R\&D$) leads to innovation that is at the core of a knowledge-based economy and of an endogenous foundation for secular growth, as shown in the second generation of growth models by Romer (1986, 1990), Lucas (1988), Rebelo (1991), and Aghion and Howitt (1992), just to name a few. However, most general equilibrium models simply ignore this aspect with respect to business cycles. This is not innocuous.[1,2]

Indeed, Comin and Gertler (2006) show that $R\&D$ effects do matter, not only for the long run, as previously thought, but also at business cycle frequencies. The exclusion of the innovation process from modern general equilibrium models deprives them from a propagation mechanism. For instance, Barlevy (2007) and Fatas (2000) both show that investments in innovation are sensitive to monetary policy shocks. The introduction of labour- and capital-augmenting innovation process would also partially endogenise technology. In conjunction with the monetary policy effect on R&D, this could even lower the contribution to business cycles that is attributed to the neutral technology shock.

In this paper, we show that the inclusion and treatment of Schumpeterian growth features matter for several reasons. First, our model highlights and addresses new challenges at the modelling and simulation stages when considering the implications of price rigidities on $R\&D$ investments. That is since the sluggish dynamics of prices impact directly on the expected discounted value derived from innovations. Second, this dimension provides some microfoundations to monopolistic competition that is introduced *de facto* in NK models, as differing levels of technological advancement in the intermediate sector bring a justification for existing market power. Third, the endogenous choice of investing in $R\&D$ has implications for the likelihood of advancing or not the technological boundary and to vary the entry and exit of firms. Therefore, the Schumpeterian dimension of our model, with Harrod-neutral technical progressing in the production function for goods and a decreasing return-to-scale innovation production function, adds a relevant transmission channel for understanding economic fluctuations and the impact of both real and monetary disturbances. In particular, we

[1] Recently, being quite critical of the widespread use of DSGE modelling in macroeconomics, Stiglitz (2018) argues that "DSGE models are, of course, not really a model of medium- to long-term growth: that is determined by factors like the pace of innovation and the accumulation of human capital on which they provide little insight. To understand the former, for instance, one needs a much more detailed analysis of technological progress, including investments in basic research and the transmission of knowledge across and within firms, than any standard macro-model can provide." As our paper shows, although challenging, it is well feasible to account for endogenous technological progress, induced by investments in $R\&D$, and transmitted by knowledge spillover.

[2] Ghironi (2018) is more confident in micro-founded DSGE models, as he argues : "If macroeconomics aims to address the dynamics of the last decade and the economic issues that have been central to recent political outcomes, artificial separations between modeling of business cycles and longer-term dynamics must be abandoned, and the same must happen to similarly artificial separations between macroeconomic and trade modeling. [...] So, yes to DSGE, and yes to micro."

consider its implications for the volatility, comovements and persistence of real variables. We believe this to be a prerequisite, prior to study the policy implications of such a model. Fourth, there are non trivial welfare implications arising from the steady-state probability value of innovating, even for a given steady-state growth rate of the technological frontier.

In this paper, we aim at jointly accounting for both endogenous growth through creative destruction and business cycles in an extended New Keynesian (NK) model. The proposed model is composed of the following groups of agents: the households, the final good producers, employment agencies, the intermediate good producers, the entrepreneurs/innovators, a monetary authority and the government. Forward-looking households maximize their expected utility with respect to their sequence of budget constraints, by making optimal decisions regarding their time paths for consumption, labour, utilization of physical capital, private investment, and net bond holdings. Final good producers operate in a perfectly competitive market and use intermediate goods as input. The employment agencies aggregate the households' specialized labour into homogeneous labour used by the intermediate good producers. The latter evolves in a monopolistically competitive setting that allows them to set prices. Operating within the intermediate sector, entrepreneurs/innovators invest final goods to increase their odds of pushing the technological frontier, so that an intermediate good producer that implements the new technology takes over the incumbent producer in his respective intermediate sector. Finally, prices and wages are subjected to nominal rigidities through contracts *à la* Calvo (1983). In particular, we show that sluggish price adjustments interact directly with the innovation process, as the discounted expected value of investing in *R&D* matters for the rate of innovation over the business cycles.

While monetary policy is set according to a Taylor rule, and deviations from thereof, the impacts of the following shocks are also considered: a transitory technological shock, a knowledge-spillover shock, and an investment shock.

By including an explicit innovation process and its interaction with the production of intermediate goods, that serve as inputs to the production of the final consumption good, we draw attention to an additional propagation mechanism for shocks. As expected, investment in *R&D* is sensitive to monetary policy shocks. A positive shock increases investments in *R&D* which, in turn, have a positive effect on marginal productivities of labour and capital. This ultimately affects the growth rates and levels of macroeconomic prices and quantities, including factor demand and wages.

Our model raises some important theoretical, methodological, and empirical points, with both qualitative and quantitative implications. Theoretically, attention had to be paid to a meticulous modelling of the interaction between Schumpeterian innovation and price rigidities, as well as to the link between business cycle and growth. Methodologically, the cyclical behaviour of the technological frontier's growth rate points out the importance of explicitly applying the same cyclical filtering to the trended simulated variables, thus requiring their construction, as well as to their empirical counterparts, as measured. Moreover, this has ramifications for the interpretation of the cyclical impulse response functions, as well as for the cumulative impact on the levels of macroeconomic variables when accounting for the cyclical response of the average growth rate of the economy. In particular, when considering the impulse response functions of the cyclical components of macroeconomic variables to various shocks, we find that the variables' dynamics are not invariant to the parameter calibration of steady-state endogenous growth.

In Section 2, we briefly review some related contributions in the literature that address related questions associated with endogenous growth and business cycles and highlight their specific emphases and features. Then, Section 3 presents in detail the attributes of our model. We begin with those that are typically encountered in modern NK-DSGE set-up, with the necessary adjustments needed to account for the presence of innovating firms. We then direct our attention to the key elements derived from the endogenous Schumpeterian considerations linked to innovations. Section 4 discusses aggregation and the model's general equilibrium, issues dealing with detrending and the steady-state properties of the model, while Section 5 deals with calibration and characterization of the various disturbances that affect our model economy. Section 6 presents and discusses some relevant statistical moments of the cyclical components of the simulated data in comparison with their empirical counterpart. Section 7 displays the impulse response functions implied by the model for the cyclical components of the macroeconomic variables and proceeds with the business cycle analysis implications. Section 8 assesses the impact on welfare associated with different steady-state values for the innovation probability and the extent of knowledge spillover, consistent with the same steady-state growth rate. A summary of our findings and possible extensions make up the conclusion in Section 9.

2 Related Literature

Until recently, relatively few papers had attempted to combine a business cycle model with the features of a Schumpeterian growth model. From the early RBC models to the most recent New-Keynesian (NK) macroeconomic models, the usual dynamic-stochastic general equilibrium (DSGE) model has indeed been constructed around a classical exogenous growth model. Indeed, seminal articles such as Smets and Wouters (2007) and Justiniano et al. (2010) feature calibrated exogenous growth.

Nuño (2011) studies a real business cycle model with Schumpeterian growth, using specific functional forms and no nominal rigidity. In a New-Keynesian model with both Calvo staggered prices and wages, as well as endogenous growth operating through non-rival access to knowledge, Annicchiarico et al. (2011) analyse monetary volatility and growth in relation with nominal rigidities, and the persistence of monetary shocks on a Taylor monetary policy rule. Amano et al. (2012) have studied the implications of endogenous growth with horizontal innovations in the variety of intermediate goods, for the welfare costs of inflation in NK economies with nominal rigidities being modelled as Taylor (1980) staggered price and wage contracts. Annicchiarico and Rossi (2013) look at optimal monetary policy in a NK economy characterized by Calvo staggered prices that embeds endogenous growth induced by knowledge externalities with increasing returns-to-scale. Annicchiarico and Pelloni (2014) examine how nominal rigidities affect uncertainty on long-term growth, when prices and wages are preset with a one-period lag, and assuming constant returns to the level of technology in the innovation activity, in a model with only labour as an endogenous input that is divided between producing output or $R\&D$. Using Bayesian estimation, Cozzi et al. (2017) show the implication of financial conditions for innovation dynamics in a NK model with price and wage rigidities arising from specific adjustment costs in a Solow-neutral technology and Schumpeterian growth. Finally, to evaluate the sources of the productivity slowdown following the Great Recession, Anzoategui et al. (2017) construct an estimate a DSGE model with staggered Calvo contracts driving sluggish adjustments of wages and final-good prices, that also features endogenous growth *via* the expanding variety of intermediate goods resulting from public learning-by-doing in the $R\&D$ process and an endogenous pace of technology adoption.

3 The Model

In this section, we describe the environment and the problems faced by each type of agents. We first describe the characteristics of the final good producer, the employment agency, and the households' problems. To a large extent, these are very similar to the standard set-up encountered in the modern dynamic stochastic general equilibrium literature. As needed, we introduce features arising from the existence of innovating firms ultimately owned by households. Second, we focus on the specificities brought by Schumpeterian considerations for the innovators and the intermediate good producers, especially with sluggish adjustment in prices. Finally, we address the aggregation issues and we present the monetary authority's policy function.

3.1 A presentation of the common features of a New Keynesian DSGE model

3.1.1 The final good producer

The final consumption good is produced by a representative firm that operates in a perfectly competitive setting and that aggregates a continuum of intermediate goods $i \in (0, 1)$ according to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} , \tag{1}$$

where Y_t is total final output, the input $Y_t(i)$ is the good produced by an intermediate level firm i, and $0 \leq \epsilon < \infty$ is the elasticity of substitution between intermediate goods.

Because of perfect competition, the final-good producer takes as given the price of its final output, P_t, and the prices of the intermediate goods, $P_t(i)$. Hence, its profit maximization problem

$$\underset{Y_t(i)}{\text{Max}}\ \Pi_{FG} = P_t\, Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di \tag{2}$$

yields the demand for the i^{th} intermediate good as a negative function of its relative price, namely

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t\ . \tag{3}$$

Since economic profits are zero under perfect competition, total nominal output is given by the sum of the nominal value of all intermediate goods i

$$P_t\, Y_t = \int_0^1 P_t(i)\, Y_t(i)\, di\ , \tag{4}$$

which, using equation (3), yields the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}}\ . \tag{5}$$

3.1.2 The employment agency

In our model economy, a continuum of households possesses different skills and offers specialized labour $L_{Ht}(j)$ for $j \in (0,1)$, that gives them some degree of market power in setting wages. Since intermediate firms use a combination of specialized labour, we can think of a representative employment agency which aggregates specialized labour and turns it into the combined labour input L_t employed by the intermediate firms, namely

$$L_t = \left(\int_0^1 L_t(j)^{\frac{\gamma-1}{\gamma}}\, dj\right)^{\frac{\gamma}{\gamma-1}}\ , \tag{6}$$

where $0 \leq \gamma < \infty$ is the elasticity of substitution between each labour type.

Operating in perfect competition, the employment agency maximizes its profits with respect to $L_t(j)$ while taking as given the aggregate wage rate W_t and the prevailing labour compensation specific to each labour type j.

The solution of its optimization problem

$$\underset{L_t(j)}{\text{Max}}\ \Pi_{EA} = W_t\, L_t - \int_0^1 W_t(j)\, L_t(j)\, dj \tag{7}$$

yields the demand for specialized labour j as a negative function of its relative wage rate

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\gamma} L_t \ . \tag{8}$$

From equation (8), and the competitive equilibrium for the employment agency, the aggregate wage rate is

$$W_t = \left[\int_0^1 W_t(j)^{1-\gamma}\, dj \right]^{\frac{1}{1-\gamma}} \ . \tag{9}$$

3.1.3 Households

The Budget Constraint

Each period t, a representative type j household faces the following budget constraint

$$P_t\, C_t \ + \ P_t\, I_t \ + \ P_t\, a(u_t)\widetilde{K}_t \ + \ \frac{B_t}{1+R_t} \ \ \leq \ \ W_t(j)\, L_t(j) \ + \ q_t\, u_t \widetilde{K}_t \ + \ B_{t-1} \ + \ D_t \ - \ T_t \ \ . \tag{10}$$

As of date t, the household's nominal value for its uses of funds is the sum of its nominal value of consumption in the final good, denoted as $P_t\, C_t$, of its desired level of investment in capital goods, $P_t\, I_t$, the resources devoted to adjust the utilization rate of physical capital (if applicable), $P_t\, a(u_t)\widetilde{K}_t$, and its end-of-period net holdings of a one-period discount bond $\frac{B_t}{1+R_t}$, where R_t the nominal interest rate between t and $t+1$.[3] It is assumed that the price of consumption, private investment in physical capital, and varying utilization of capital is that of the aggregate price level P_t.

Our set-up allows a time-varying utilization of the existing stock of physical capital, $\widetilde{K}_t$, lower than 100%, provided that the household bears a cost of varying capital utilization u_t. This real cost is captured by a convex function $a(u_t)$ that is increasing in u_t.[4]

Type j household's nominal after-tax sources of funds arise from its labour income, i.e. the product of its nominal wage rate $W_t(j)$ and hours worked $L_t(j)$, its nominal payments received from supplying capital services to intermediate firms, from renting some of its existing physical capital, $u_t\ \widetilde{K}_t$, at a gross capital rental rate q_t, the nominal face value of the net discount bond holdings carried from the previous period, and the nominal dividends, D_t, received from its ownership of shares in the intermediate production sector that operates in monopolistic competition, from which is subtracted the value of lump-sum taxes, net of government transfers.[5]

[3] The net bond holdings may be positive or negative depending on whether the household is either creditor or debtor. Yet, for this closed economy, the aggregate net bond holdings is zero in equilibrium.

[4] It is also assumed that, in the special case where the capital stock is used at full capacity with $u_t = 1$, this real cost function takes a value of zero, i.e. $a(1) = 0$.

[5] It is assumed that the government follows a Ricardian fiscal policy.

We, therefore, need to assess the dividends stemming from the economic rent, as implied in part by investments in *R&D*. Using aggregate labour and capital, a firm i, belonging to a continuum defined over $i \in (0,1)$, produces intermediate good i in a monopolistically competitive market, thus generating positive economic profits

$$\Pi_{i,t} = P_t(i)\,Y_t(i) - w_t\,L_t(i) - q_t\,K_t(i)\ , \tag{11}$$

that are in turn paid as dividends among households.

The investment in *R&D* has to be accounted for in each period, while being treated as a sunk cost since it is irrelevant whether or not an innovator is successful *ex-post*. We will use two examples to illustrate how we account for the innovation process in the representative household's budget constraint.

Having invested $P_t\,X_t$ to reach the frontier, a failed innovator generates no profits, as depicted on the timeline of his cash flow in Figure 1.

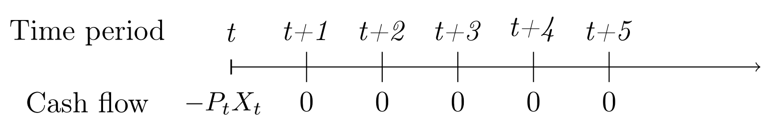

Figure 1: A failed innovator's timeline for cashflows

In this case, the household's budget constraint needs only to include his initial investment. The *ex-post* value of engaging in the innovation process is $-P_tX_t$.

By comparison, a successful innovator invests $P_t\,X_t$ in *R&D*, which turns into a successful endeavour that allows him to collect monopoly profits π_t each period, for τ periods, until it is replaced by a new innovator. Figure 2 illustrates the corresponding flow timeline.

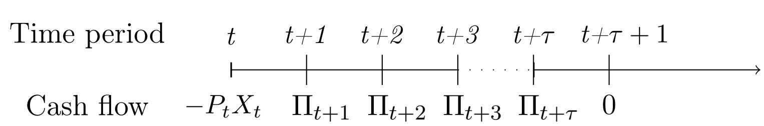

Figure 2: A succesfull innovator's timeline for cashflows

Here, the initial investment as well as future profits should be included in their respective budget constraints. It is important, however, to highlight that the profits included in the timeline above do not exclusively result from the innovation process. Indeed, an intermediate firm is already generating profits prior to an innovator taking over. Hence, the profits generated by the intermediate firm after the takeover includes both monopoly and innovation profits.

Accordingly, the overall dividends paid to households are therefore defined as:

$$D_t = \int_0^1 \Pi_{i,t} - P_t\, X_t(i)\, di \ . \tag{12}$$

Utility maximization

Similarly to Christiano et al. (2005), household j maximizes its utility function with respect to the sequence of its budget constraints for each period, while taking into account the law of movement of capital. Its preferences for consumption embed habit formation, with an intensity parameter $h > 0$, which generates some additional intrinsic dynamics and persistence on both the demand and supply sides of the economy following various shocks. The subjective discount factor is $0 < \beta < 1$, the parameter $\theta > 0$ induces disutility of labour, and the parameter $\nu \geq 0$ implies that the Frisch elasticity of labour supply is $1/\nu$. Furthermore, we assume that the household incurs some cost of adjusting investment $S(\cdot)$, that is an increasing concave function of the growth rate in investment.

The representative household must decide how much to consume C_t, while allowing for some habit formation, how many hours $L_t(j)$ to work, how much capacity to use u_t, how much physical capital they want next period $\widetilde{K}_{t+1}$, how much to invest I_t in physical capital, and the size of their net bond holdings B_t by solving the following optimization problem:

$$\underset{C_{t+s},L_{t+s}(j),u_{t+s},\tilde{K}_{t+s+1},I_{t+s},B_{t+s}}{\text{Max}} E_t^j \sum_{s=0}^{\infty} \beta^s \left(ln\big(C_{t+s} - h\, C_{t+s-1}\big) - \theta\, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right) \tag{13}$$

subject to

$$P_{t+s}\, C_{t+s} + P_{t+s}\, I_{t+s} + P_{t+s}\, a(u_t)\widetilde{K}_{t+s} + \frac{B_{t+s}}{1+i_t} \leq W_{t+s}(j)\, L_{t+s}(j) + q_{t+s}\, u_{t+s}\widetilde{K}_{t+s} + B_{t+s-1} + D_{t+s} - T_{t+s} \ , \tag{14}$$

$$\widetilde{K}_{t+s+1} = \mu_{I,t+s} \cdot \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} + (1-\delta)\widetilde{K}_{t+s} \ , \tag{15}$$

$$\ln \mu_{I,t+s} = \rho_I\, \ln \mu_{I,t+s-1} + \epsilon_{I,t+s} \tag{16}$$

$$K_{t+s} = u_{t+s}\widetilde{K}_{t+s} \ . \tag{17}$$

where E_t^j is the expectation operator conditioned of known information as of the beginning of period t. The function $S(\,\cdot\,)$ represents a convex adjustment function cost incurred when transforming current and past investment into installed capital.[6] Moreover, an exogenous stochastic investment shock $\mu_{I,t+s}$, that affects the efficiency with which investment is transformed into capital, follows a first-order autoregressive process.

In addition, having assumed that a household j possesses some specialized skills underlying some market power over its wage rate, we also assume the existence of wage rigidities modelled with Calvo contract arrangements, with a constant proportion $1-\xi_w$ being allowed to reoptimize their wage each period. Hence, household j maximizes its expected utility weighed by the probability ξ_w of not being allowed to optimize with respect to wages subject to the labour demand function:

$$\underset{W_{t+s}(j)}{\text{Max}}\; E_t^j\left(\sum_{s=0}^{\infty}\xi_w^s\,\beta^s\left(-\,\theta\,\frac{L_{t+s}(j)^{1+\nu}}{1+\nu}\right)+\Lambda_{t+s}\,W_{t+s}(j)\,L_{t+s}(j)\right) \tag{18}$$

subject to

$$L_{t+s}(j)=\left(\frac{W_t(j)\,\pi_{w\,t,t+s}}{W_{t+s}}\right)^{\gamma}L_{t+s}\ , \tag{19}$$

where γ is type-j labour demand elasticity to the relative wage, and $\pi_{w\,t,t+s}$ is the gross rate of backward wage indexation from t and $t+s$, built in the wage-setting rule.

Accordingly, the optimal reset wage is obtained from

$$W_t^*(j)^{-\gamma\nu-1}=\frac{\gamma-1}{\theta\gamma}\frac{\sum\limits_{s=0}^{\infty}\xi_w^s\beta^s\Lambda_{t+s}(\pi_{w\,t,t+s})^{1-\gamma}W_{t+s}^{\gamma}L_{t+s}}{\sum\limits_{s=0}^{\infty}\xi_w^s\beta^s(\pi_{w\,t,t+s})^{-\gamma(1+\nu)}W_{t+s}^{\gamma(1+\nu)}L_{t+s}^{1+\nu}}\ . \tag{20}$$

Exploiting the relevant recursions built in the summation, which, in turn, will be useful for subsequent numeric simulation, equation (20) can be rewritten as

$$W_t^*(j)^{-\gamma\nu-1}=\frac{\gamma-1}{\theta\gamma}\frac{Aux_{bc,t}}{Aux_{dis,t}}\ , \tag{21}$$

where, we define auxiliary variables associated respectively with the household's budgets constraint in the numerator, $Aux_{bc,t}$, and the disutility of labour in the denominator, $Aux_{dis,t}$, i.e.

$$Aux_{bc,t}=\Lambda_t\,W_t^{\gamma}L_t+\xi_p\beta(\pi_{w\,t,t+1})^{1-\gamma}Aux_{bc,t+1}\ , \tag{22}$$

[6] At the calibration stage, we assume it to be defined as $S\big(I_t/I_{t-1}\big)=(\kappa/2)\,\bullet\big(I_t/I_{t-1}-g_t\big)^2$.

and

$$Aux_{dis,t} = \Lambda_t W_t^{\gamma(1+\nu)} L_t^{1+\nu} + \xi_p \beta (\pi_{w\,t,t+1})^{-\gamma(1+\nu)} Aux_{dis,t+1} \ . \tag{23}$$

3.2 The Schumpeterian add-ons in a New-Keynesian DSGE model and their implications

Building on Aghion and Howitt (1992) and Nuño (2011), our Schumpeterian growth paradigm assumes that an innovation may arise from investing in $R\&D$ in period $t-1$ and push the technology level at A_{t-1}^{max} for an intermediate firm operating at date t with an endogenous probability n_{t-1}, making it an advanced firm. Otherwise, there is a $1-n_{t-1}$ probability that an intermediate firm is lagging, while still using older technology level.

However, the existence of price stickiness adds an additional level of complexity that is absent in typical Schumpeterian growth models, as well as in recent business cycles models with Schumpeterian features without price rigidity. Namely, since a firm's investments in $R\&D$ depends on the discounted expected profits arising from innovating, if successful, then the monopolistic rent also depends on the expected prices that a firm will be allowed to set (with some probability that prices may be fixed, and some probability that they may be adjusted).

While implementing a newly discovered innovation, we assume that an intermediate firm is allowed to set the optimal price right away. However, at subsequent dates (quarters), barring some new innovation, the same intermediate firm is stuck with a more or less older technology, and there are probabilities that its price remains sticky for some time. Indeed, if it is not innovating, the lagging firms are in a Calvo-type environment as in standards NK models.

Hence, three categories of intermediate firms are coexisting. Advanced firms which reset the optimal price, lagging firms which are allowed to reoptimize their respective price, and lagging firms with previously set prices.

3.2.1 The optimal reset price

Operating in a monopolistically competitive market, intermediate firms hold market power from both their diversification and the technology used in production. In itself, it is worth noticing that the mechanics of innovation being considered also brings some support and microfoundations to the monopolistic competition *de facto* introduced in the usual New Keynesian models. Moreover, prices are fixed through Calvo contracts and set as to maximize their expected profits conditional on not being allowed to optimize.

Hence, given an initial level of technological advancement $A_t(i)$, an intermediate firm faces the following constrained minimization of their cost:

$$\underset{K_t(i),L_t(i)}{\text{Min}} \; W_t L_t(i) + q_t K_t(i) \tag{24}$$

subject to

$$Y_t(i) = \mu_{Z,t}\, A_t(i)^{1-\alpha}\, K_t(i)^{\alpha}\, L_t(i)^{1-\alpha} \;, \tag{25}$$

where

$$\ln \mu_{Z,t} = \rho_Z \; \ln \mu_{Z,t-1} + \epsilon_{Z,t} \;, \tag{26}$$

so that, regardless of their individual level of technological advancement, all intermediate firms' productions are subjected to a common transitory technological shock, that follows a first-order autoregressive process.

Accordingly, the optimal capital-labour ratio (that is identical for all intermediate firms) being employed is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{q_t} \;, \tag{27}$$

and the nominal marginal cost of producing an additional unit of intermediate good is given by

$$MC_t(i) = A_t(i)^{\alpha-1}\, \Omega_t \tag{28}$$

where Ω_t is the portion of marginal costs that is not directly dependent of the level of technology, i.e.

$$\Omega_t = \frac{q_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \;. \tag{29}$$

Hence, an improvement in technology leads to a lowering of a firm's marginal cost.

In traditional New-Keynesian models, all firms operate at the same level of technological advancement, so that, when possible, all firms set the same optimal reset price. In contrast, since the optimal reset price depends on the technology, in our set-up, with an infinite number of intermediate firms, there is an infinite number of coexisting technologies. Accordingly, there is an infinite number of reset prices because the marginal cost is a function of the technology level.

Consequently, given their respective marginal cost, intermediate firms maximize their profits with respect to their price $P_t(i)$:

$$\underset{P_t(i)}{\text{Max}} \; E_t\left\{ \sum_{s=0}^{\infty} \xi_p^s\, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[n_{t+s-1} \Big(P_t(i)\pi_{p\,t,t+s} - MC_{t+s}(i) \Big)\, Y_{t+s}(i) \right] \right\} \tag{30}$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t(i)\, \pi_{p\,t,t+s}}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \ , \tag{31}$$

where $\pi_{p\,t,t+s}$ is the gross rate of backward price indexation from t and $t+s$, built in the price-setting rule. To solve this problem, it must be noticed that initial technology is known and can differ between firms, while future technology levels depend as well on future investment levels in $R\&D$.

Thus, it can be shown that the optimal reset price is a function of initial technology $A_t(i)$, and of a factor F_t that is not directly dependent of the technology level:

$$P_t^*(i) = A_t(i)^{\alpha-1} F_t \ , \tag{32}$$

where

$$F_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{s=0}^{\infty} \xi_p^s \, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} (\pi_{p\,t,t+s})^{-\epsilon} \Omega_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{\sum_{s=0}^{\infty} \xi_p^s \, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} (\pi_{p\,t,t+s})^{1-\epsilon} P_{t+s}^{\epsilon} Y_{t+s}} \ . \tag{33}$$

Then, for algebraic elegance and later computational convenience, the involved summations in equation (33) can be expressed in a recursive form, so that

$$F_t = \frac{\epsilon}{\epsilon - 1} \frac{Aux_{cost,t}}{Aux_{rev,t}} \ , \tag{34}$$

where

$$Aux_{rev,t} = P_t^{\epsilon} Y_t + \xi_p \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1 - n_t) (\pi_{p\,t,t+1})^{1-\epsilon} Aux_{rev,t+1} \ , \tag{35}$$

and

$$Aux_{cost,t} = \Omega_t P_t^{\epsilon} Y_t + \xi_p \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1 - n_t) (\pi_{p\,t,t+1})^{-\epsilon} Aux_{cost,t+1} \ . \tag{36}$$

3.2.2 The innovation process

$R\&D$ activities are conducted by entrepreneurs/innovators. If these lead to an innovation, the implementation of this technology by an intermediate good producer conveys some additional market power from producing an improved version of the intermediate good, as it reaches the new technological frontier. Hence, the innovation process unfolds within the intermediate sector as the mechanism that pushes the technological frontier outward.

An entrepreneur/innovator invests some amount of final goods to raise the probability of innovating. Outside researchers or a new successful innovator supplants or "leapfrogs" an incumbent entrepreneur.[7] This prospect is, however, uncertain as the probability to innovate is n_t, and that of not discovering is $1 - n_t$. Yet, n_t is endogenous as it is linked to the intensity of $R\&D$ effort $\frac{X_t}{\zeta A_t^{max}}$, where X_t is the real amount of final goods invested in $R\&D$, A_t^{max} is the targeted technology level, or frontier, that will be used in date t+1 production, and $\zeta > 1$ is a scaling factor that can be derived from an innovation. In particular, when A_t^{max} is bigger, a given amount of resources X_t in $R\&D$ is associated with a lower intensity, thus capturing the increasing complexity of further progress. Finally, we consider the following innovation production function that exhibits diminishing marginal returns, with $\eta > 0$:

$$n_t = \left(\frac{X_t}{\zeta A_t^{max}} \right)^{1/(1+\eta)} \quad . \tag{37}$$

Furthermore, the gross growth rate of the technological frontier A_t^{max} is dictated by the probability of innovation times a spillover factor, which is subject to some first-order autoregressive stochastic component. This defines a proportional increase in productivity resulting from an innovation. Namely,

$$A_t^{max} = g_t^{max} \cdot A_{t-1}^{max} , \tag{38}$$

$$g_t^{max} = 1 + \sigma_t n_{t-1} , \tag{39}$$

where

$$\ln \sigma_t = \ln \sigma + \rho_\sigma \ln \sigma_{t-1} + \epsilon_{\sigma,t} . \tag{40}$$

The introduction of a stochastic spillover shock on σ_t can account for unpredictable variations and other heterogeneities in the transmission of knowledge and/or abilities to capitalize on new innovations to push the technological frontier further. Hence, a knowledge spillover is the positive externality derived from an innovation, since it permanently pushes the technological frontier forward.[8] The higher the value of σ_t, the greater is the extent of technology spillover, which, in turn, induces a larger technological jump of the frontier.

[7]In our model, we therefore abstract from step-by-step technological progress, that would imply both Schumpeterian and escape-competition effects. This can also be justified by assuming that, from engaging in $R\&D$, it is prohibitively costly to develop a perfectly substitutable technology that can be used to produce a cheap and faked copy of an existing intermediate good, i.e. a knockoff.

[8]Baldwin et al. (2005) have shown, both theoretically and empirically, how knowledge spillovers generated by multinational corporations and foreign direct investments enhance growth endogenously. It is natural to argue that a similar effect can span across firms within an industry, as well as across industries to some extent. For instance, we can think of spillovers from the diffusion in information and communication technologies. Moreover, based on industry level data for 15 OECD countries, Saia et al. (2015) recently showed the significance of knowledge spillovers for an economy's effectiveness to learn from the technological frontier and to increase productivity. An economy's spillovers stem from "its degree of international connectedness, ability to allocate skills efficiently and investments in knowledge-based capital, including managerial capital and $R\&D$."

To choose the amount of final good to be invested in $R\&D$ that maximizes expected discounted profits, the entrepreneur faces the following constrained optimization problem

$$\underset{X_t}{\text{Max}}\ \beta \frac{\Lambda_{t+1}}{\Lambda_t} n_t E_t V_{t+1}(A_t^{max}) - P_t X_t \tag{41}$$

subject to equation (37), where $E_t V_{t+1}(A_t^{max})$ is the expected discounted value of future profits contingent on the entrepreneur remaining at the helm of the monopoly. If successful, the innovator will collect monopoly profits as long as no further innovation occurs in his sector.[9]

For convenience and in a way that is compatible with complete markets, we assume that entrepreneurs invest in a diversified form of $R\&D$. Strictly speaking, this is as if an entrepreneur, provided one is successful, does not know in which sector, one may end up. This implies that all entrepreneurs will invest the same amount of final good in $R\&D$. Consequently, if an innovation occurs, the technology jumps to the frontier and the expected value of the intermediate firm will be the same regardless of the intermediate sector. In accordance with the problem in equation (41), the optimal real investment in $R\&D$ is given by

$$X_t = \beta \frac{n_t}{1+\eta} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{E_t V_{t+1}(A_t^{max})}{P_t} \ . \tag{42}$$

To complete the solution to equation (42), we still need to write explicitly the expected value of the firm to the entrepreneur. This happens to be more challenging than it may look at first, as it relies on the path that prices are expected to follow.

The first hurdle is to ensure that the assessment of future profits follows the correct technological path. For instance, an innovator may reach the frontier A_t^{max} in $t+1$, and remains at that level, say until $t+3$, as it is supplanted by an advanced firm, with a lower marginal cost of operations. In this situation, from then on, the following expected profits no longer matter for investing in innovation since it is now out of business.

The second difficulty springs from price rigidities that play a crucial role in determining future profits because they condition both the profit margin and the conditional demand for that specific intermediate good. Innovative intermediate firms that jump to the frontier are automatically allowed to reoptimize. Non-innovating ones may be selected for reoptimization using Calvo contracts, with

[9] In our model, with a steady-state probability of innovation n^{ss}, the expected life span of an intermediate firm at steady-state is $1/n^{ss}$.

a probability ξ_p of not being allowed to optimize their price. Because we assume that investments in $R\&D$ are diversified by the entrepreneurs/innovators, even though, *ex-ante*, an entrepreneur ignores the sector in which he will be innovating, he can evaluate the discounted expected profits from a potential discovery using the individual prices of the continuum of intermediate goods.

To reckon the value of an innovation-implementing intermediate firm, let us consider an entrepreneur/innovator who, at date t, ponders how much to invest in $R\&D$ while seeking some returns from date $t+1$ onward. This evaluation must take into account different possible outcomes that reflect the probability of remaining at the helm of the monopoly, the appropriate stochastic discount factors, as well as the probability that price reoptimization occurs under Calvo contracts. This is why, we have to consider all possible contingencies that could deliver some return from innovating.[10]

First, let us think about the case of a new monopolist taking over as of date $t+1$ and setting the optimal price for its intermediate good i. As he has innovated, his prevailing specific technology reaches the new technological frontier, so that $A_t(i) = A_t^{max}$. He faces each period the probability ξ_p of not being allowed to optimize its price afterwards, so that, for the contingency path that price reoptimization never occurs, his expected discounted stream of profits would be given by

$$\Psi_{1t+1}(i) = \sum_{s=1}^{\infty} \xi_p^{s-1}\, \beta^{s-1}\, \frac{\Lambda_{t+s}}{\Lambda_{t+1}} \left[P_{t+1}^{*}(i)\pi_{p\,t,t+s} - MC_{t+s}(i)\right] Y_{t+s}(i) \prod_{q=2}^{s} (1 - n_{t+q-1}) \ , \quad (43)$$

where $\prod_{q=2}^{s}(1-n_{t+q-1})$, is the probability of not being displaced out of business at date $t+s$. The flows of revenues and costs are discounted from the perspective of date $t+1$, as the nested sum is discounted up to the beginning of the initial cash flow pertaining to this stream, and weighted by the probability of remaining at the helm of the monopoly for all periods in the future. In particular, date $t+1$ cash flow has a unit probability, i.e. $\prod_{q=2}^{1}(1-n_{t+q-1}) = 1$, as we consider a successful innovation driving the production of intermediate good i that is sold at its optimal price. Moreover, if an intermediate firm producing good i is replaced following the implementation of a new innovation in this sector at some future date T, then no additional profits will accrue from then on.

As long as it has not been supplanted, this monopolist will be operating under technology A_t^{max}. Hence, his marginal cost of production evolves according to

$$MC_{t+s}(i) = A_t^{max\,(\alpha-1)}\, \Omega_{t+s} \ , \quad (44)$$

while his optimal price is set to

$$P_{t+1}^{*}(i) = A_t^{max\,(\alpha-1)}\, F_{t+1} \ , \quad (45)$$

[10]Notice that if the monopolist is supplanted at a future date by a competitor's adoption of an innovation, expected profits are to become zero from that date forward, with no bearing on the current expected discounted flow of profits for date t investing entrepreneur.

and the expected demand for his good follows a path defined by

$$Y_{t+s}(i) = \left(\frac{P^*_{t+1}(i)\, \pi_{p\,t,t+s}}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \ . \tag{46}$$

Using equations (44), (45), and (46), equation (43) can be rewritten as

$$\begin{aligned} \Psi_{1t+1}(i) = \quad & A_t^{max\,(\alpha-1)(1-\epsilon)} \left\{ F_{t+1}^{(1-\epsilon)} \sum_{s=1}^{\infty} \xi_p^{s-1}\, \beta^{s-1} \frac{\Lambda_{t+s}}{\Lambda_{t+1}}\, P^{\epsilon}_{t+s} \pi^{(1-\epsilon)}_{p\,t+1,t+s} Y_{t+s} \prod_{q=2}^{s} (1 - n_{t+q-1}) \right. \\ & \left. - F_{t+1}^{-\epsilon} \sum_{s=1}^{\infty} \xi_p^{s-1}\, \beta^{s-1} \frac{\Lambda_{t+s}}{\Lambda_{t+1}}\, P^{\epsilon}_{t+s} \pi^{-\epsilon}_{p\,t+1,t+s} \Omega_{t+s} Y_{t+s} \prod_{q=2}^{s} (1 - n_{t+q-1}) \right\} , \end{aligned} \tag{47}$$

Making use of equations (35) and (36), but as of $t+1$, we therefore have

$$\Psi_{1t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)} \left(F_{t+1}^{(1-\epsilon)} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1} \right) , \tag{48}$$

with auxiliary variables associated respectively with the firm's revenues in the first term, $Aux_{rev,t}$, and costs in the second term, $Aux_{cost,t}$, i.e.

$$Aux_{rev,t+1} = P^{\epsilon}_{t+1} Y_{t+1} + \xi_p \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 - n_{t+1}) (\pi_{p\,t+1,t+2})^{1-\epsilon} Aux_{rev,t+2} \ , \tag{49}$$

and

$$Aux_{cost,t+1} = \Omega_{t+1} P^{\epsilon}_{t+1} Y_{t+1} + \xi_p \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 - n_{t+1}) (\pi_{p\,t+1,t+2})^{-\epsilon} Aux_{cost,t+2} \ . \tag{50}$$

Second, let us turn to all the other possible cases of a new monopolist taking over as of date $t+1$ and setting the optimal price for its intermediate good i at some future date $t+l$ with some probability $1-\xi_p$, yet followed by the contingency path that price reoptimization does not occur afterwards, as there is a probability ξ_p each period of no reoptimization, even if the monopolist remains in operation. Summing over all contingent paths, with the proper probabilistic weights, the relevant discounted stream of profits for all these contingent paths is given by

$$\begin{aligned} \Psi_{2t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)} & \left\{ \sum_{l=2}^{\infty} (1-\xi_p) \beta^l \frac{\Lambda_{t+l}}{\Lambda_{t+1}} \prod_{q=2}^{l-1} (1 - n_{t+q-1}) \right. \\ & \cdot \left[F_{t+l}^{1-\epsilon} \sum_{s=l}^{\infty} \xi_p^{s-1} \beta^{s-1} \frac{\Lambda_{t+s}}{\Lambda_{t+l}} (\pi_{p\,t+l,t+s})^{1-\epsilon} P^{\epsilon}_{t+s} Y_{t+s} \prod_{q=l}^{s} (1 - n_{t+q-1}) \right. \end{aligned} \tag{51}$$

$$-F_{t+l}^{-\epsilon}\sum_{s=l}^{\infty}\xi_p^{s-1}\,\beta^{s-1}\,\frac{\Lambda_{t+s}}{\Lambda_{t+l}}\,P_{t+s}^{\epsilon}\pi_{p\,t+1,t+s}^{-\epsilon}\Omega_{t+s}Y_{t+s}\,\prod_{q=l}^{s}(1-n_{t+q-1})\Big]\Bigg\}\ .$$

Furthermore, updating equations (49) and (50) to date $t+l$, equation (51) can be rewritten as

$$\Psi_{2t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)} \tag{52}$$

$$\cdot\left\{\sum_{l=2}^{\infty}(1-\xi_p)\beta^l\frac{\Lambda_{t+l}}{\Lambda_{t+1}}\prod_{q=2}^{l-1}(1-n_{t+q-1})\left(F_{t+l}^{(1-\epsilon)}Aux_{rev,t+l}-F_{t+l}^{-\epsilon}Aux_{cost,t+l}\right)\right\}$$

In an analogous manner as before, making use of the recursion built in the summation above and defining an auxiliary variable, $Aux_{rem,t+l}$, associated with the remainder of the expected discounted profits equation (52) can also be displayed as

$$\Psi_{2t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)}\cdot Aux_{rem,t+2} \tag{53}$$

where

$$Aux_{rem,t+2} = (1-\xi_p)\beta^2\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\left(F_{t+2}^{(1-\epsilon)}Aux_{rev,t+2}-F_{t+2}^{-\epsilon}Aux_{cost,t+2}\right)$$

$$+\beta\left(1-n_{t+2}\right)Aux_{rem,t+3}\ . \tag{54}$$

Consequently, the expected value of the intermediate firm to a successful innovator is

$$E_t\,V_{t+1} = \Psi_{1t+1}(i)+\Psi_{2t+1}(i) \tag{55}$$

or, namely,

$$E_t\,V_{t+1} = A_t^{max\,(\alpha-1)(1-\epsilon)}\cdot\left(F_{t+1}^{1-\epsilon}Aux_{rev,t+1}-F_{t+1}^{-\epsilon}Aux_{cost,t+1}\,+\,Aux_{rem,t+2}\right)\ . \tag{56}$$

Hence, the expected value of an intermediate firm for a successful entrepreneur/innovator is determined by the newly reached technological frontier through $A_t^{max\,(\alpha-1)(1-\epsilon)}$, the contribution from profits arising from being able to set the optimal price for good i as of period $t+1$ through $F_{t+1}^{1-\epsilon}Aux_{rev,t+1}-F_{t+1}^{-\epsilon}Aux_{cost,t+1}$, and the contribution to profits resulting from a later date optimal price setting with what will have become an older technology through $Aux_{rem,t+2}$.

3.3 The specification of monetary policy

The central bank's policy function is modelled as a Taylor-type reaction function, as it sets the nominal interest rate according to the following equation:

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\, g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R} \mu_{M,t} \ , \tag{57}$$

where

$$\ln \mu_{M,t} = \rho_M \ \ln \mu_{M,t-1} + \epsilon_{M,t} \ . \tag{58}$$

The ρ_R parameter represents the degree of smoothing of interest rate changes, as the monetary authority seeks to avoid too large changes with respect to its one-period-lag value, R_{t-1}, and adjusts it somewhat gradually, with weight $1-\rho_R$, following demand and technology shocks. The parameters α_π and α_y are the monetary authority's weights attached to deviations from its inflation target, π, and its output growth target, g. The latter is defined as the growth rate of the average technology level in steady state. Finally, $\mu_{M,t}$ is an exogenous and stochastic component of monetary policy representing deviations from the Taylor-type rule, that follows a stationary first-order autoregressive process.

4 The Aggregate Economy

We now turn to the aggregation of the key economic variables and the equilibrium conditions.

4.1 The aggregate price level

From equation (5), the overall economy's aggregate price level can be inferred by weighting and combining each of the respective prices for the three categories of coexisting firms

$$\begin{aligned} P_t^{1-\epsilon} \ &= \ \xi_p (1-n_{t-1})(P_{t-1}\pi_{p\,t-1,t})^{1-\epsilon} \\ &\quad + \ (1-\xi_p) \int_{n_{t-1}}^{1} P_t^*(i)^{1-\epsilon} di \ + \int_0^{n_{t-1}} P_t^*(i)^{1-\epsilon} di \ . \end{aligned} \tag{59}$$

The first term above corresponds to firms operating some older technologies that are not allowed to reset their prices, yet applying backward price indexation up to t. The second term is associated to firms using older technologies that can reset their optimal price. Finally, the last group is that of new monopolists adopting the most recent technology and accordingly setting the intermediate good's optimal price.

In a conventional New Keynesian model in which long-term growth is exogenous, the price index includes both prices indexed to inflation and optimal reset prices. However, in an endogenous growth environment, we have to consider separately and explicitly, both reoptimizing-firms types: those that have innovated and those that have not. Innovating firms all set the same optimal reset price, as they are automatically allowed to reoptimize. Lagging firms set different optimal reset prices according to their respective level of technological advancement. The difference between these two indices, as captured by the third term on the right-hand side of equation (59), is related to what Aghion et al. (2018) identify as missing growth. As they point out, a measurement error in inflation stems from leaving the impact of innovative goods out. In turn, this error has consequential implications for the effective real output growth rates. Since real output growth is obtained by subtracting inflation from nominal output growth, an overestimation of inflation leads to an underestimation of output growth. This is likely to have implications for monetary policy, as well as for the design of subsidization and tax-credit policies conducive to $R\&D$, that could be considered in future work.

Exploiting the implicit recursion embedded in equation (59), we can show that

$$\begin{aligned} P_t^{1-\epsilon} \;=\; & \xi_p(1-n_{t-1})(P_{t-1}\pi_{p\,t-1,t})^{1-\epsilon} \\ & + \;(1-\xi_p)(1-n_{t-1})F_t^{1-\epsilon}\,Aux_{oldtech,t-1} \;+\; n_{t-1}A_{t-1}^{max\,(\alpha-1)(\epsilon-1)}F_t^{1-\epsilon} \end{aligned} \tag{60}$$

where

$$Aux_{oldtech,t-1} \;=\; n_{t-2}\,A_{t-2}^{max\,(\alpha-1)(1-\epsilon)} \;+\; (1-n_{t-2})\,Aux_{oldtech,t-2} \tag{61}$$

The auxiliary variable, denoted by $Aux_{oldtech,t-1}$, is itself a weighted average of the older technology levels prevailing as of date $t-1$.

4.2 The aggregate wage rate

From the aggregate wage index, defined by equation (9), the overall economy's wage index can be inferred from weighting the respective wages for the workers that, yet applying backward wage indexation up to t, but cannot reset their wage optimally, and those that are allowed to reset to the optimal wage, i.e.

$$W_t^{1-\gamma} \;=\; \xi_w\,(W_{t-1}\,\pi_{w\,t-1,t})^{1-\gamma} + (1-\xi_w)\,W_t^{*\,1-\gamma}\;. \tag{62}$$

4.3 Aggregate output

The aggregation of output needs to account for the fact that many intermediate firms coexist with different technology levels and, hence, specific output levels.

From equations (3) and (25), considering that the optimal capital-labour ratio is identical for all firms, and integrating over all the firms on the $[0,1]$ continuum, we can show that

$$\mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} = P_t^{\epsilon} Y_t \int_0^1 A_t(i)^{\alpha-1} P_t(i)^{-\epsilon} di \tag{63}$$

Since three classes of situations arise, the integral in equation (63) can be accordingly evaluated over three subintervals:

$$\begin{aligned} \mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} \;=\; & P_t^{\epsilon} Y_t \cdot \Bigg[\int_0^{\xi_p(1-n_{t-1})} A_t(i)^{\alpha-1} P_t(i)^{-\epsilon} di \\ & + \int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{\alpha-1} P_t^*(i)^{-\epsilon} di + \int_{1-n_{t-1}}^{1} A_t(i)^{\alpha-1} P_t^*(i)^{-\epsilon} di \Bigg] . \end{aligned} \tag{64}$$

The first and second intervals include old-technology-running firms that are respectively non resetting, and optimally resetting their price at $P_t^*(i)$. The last one covers the innovating firms with optimal price setting at $P_t^*(i)$. Using equation (32), the above equation becomes

$$\begin{aligned} \mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} \;=\; & P_t^{\epsilon} Y_t \cdot \Bigg[(P_{t-1} \pi_{p\,t-1,t})^{-\epsilon} \int_0^{\xi_p(1-n_{t-1})} A_t(i)^{\alpha-1} di \\ & + F_t^{-\epsilon} \int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{(1-\epsilon)(\alpha-1)} di + F_t^{-\epsilon} \int_{1-n_{t-1}}^{1} A_t(i)^{(1-\epsilon)(\alpha-1)} di \Bigg] \end{aligned} \tag{65}$$

Then, since the firms involved within the first and second integrals operate older technologies, since they do not innovate, the distribution of technologies in these sectors is identical to that of the entire economy as it was one period before. Accordingly, we can define auxiliary variables that can be used to evaluate the relevant integrals by exploiting corresponding recursions. Namely, we can show that:

$$\begin{aligned} \int_0^{\xi_p(1-n_{t-1})} A_t(i)^{(\alpha-1)} di \;&=\; Aux_{oldtechnonreset,t-1} \\ &=\; \xi_p(1-n_{t-1})\, A_{t-2}^{max\,(\alpha-1)} + (1-n_{t-2}) Aux_{oldtechnonreset,t-2}\ , \end{aligned} \tag{66}$$

and

$$\int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{(1-\epsilon)(\alpha-1)} di = (1-\xi_p)(1-n_{t-1}) Aux_{oldtech,t-1}\ . \tag{67}$$

At last, the firms that belong to the third integral have innovated and pushed the frontier as of last period, so that

$$\int_{1-n_{t-1}}^{1} A_t(i)^{(1-\epsilon)(\alpha-1)} di \;=\; A_t^{max\,(1-\epsilon)(\alpha-1)}\ . \tag{68}$$

Referring to equations (67),(67), and (68), it follows that aggregate output must satisfy

$$P_t^{\epsilon} Y_t \; = \mu_{Z,t} \, \frac{K_t^{\alpha} L_t^{1-\alpha}}{Aux_{output,t}} \; , \tag{69}$$

where

$$\begin{aligned} Aux_{output,t} \; = \; & \xi_p (1 - n_{t-1})(P_{t-1}\pi_{p\,t-1,t})^{-\epsilon} Aux_{oldtechnonreset,t-1} \\ & + (1-\xi_p)(1-n_{t-1}) F_t^{-\epsilon} Aux_{oldtech,t-1} \\ & + n_{t-1} F_t^{-\epsilon} (A_{t-1}^{max})^{(1-\epsilon)(\alpha-1)} \end{aligned} \tag{70}$$

4.4 The aggregate resource constraint

Accounting for investment in $R\&D$, the aggregate resource constraint satisfies

$$C_t \, + \, I_t \, + \, a(u_t)\widetilde{K}_t \, + \, X_t \, = \, Y_t \; , \tag{71}$$

since the sum of private consumption, investment, resources devoted to adjust the utilization rate of capital, and investment in $R\&D$ is bounded by the production of final goods.

4.5 General equilibrium and key Euler equations for the model to be solved

The general equilibrium of the model requires that supply and demand be equated for the final goods market, the intermediate goods market, the credit (bond) market, the labour market, the markets for physical capital, for investment in physical capital, as well as in $R\&D$. Appendix A collects all the relevant equations that need to be satisfied.

4.6 Detrending and model solution

In this model, output, consumption, physical capital, investments in physical capital and investments in research and development fluctuate around a balanced growth path because of technological growth. The existence of the balanced growth path is ensured by the use of labour augmenting technology.

The variables need to be detrended before simulating the model around the steady state. Detrending can be done either by dividing aggregate variables by the technological frontier or the average technology. Investments in $R\&D$ being a function of the technological frontier one could argue that detrending by the technological frontier is the correct way to do it. However, because aggregate output, investment and consumption are functions of the average prevailing technology level, we can also detrend by the average technology. In the context of an exogenous growth model, this question would be irrelevant since the frontier and the average technology are one and the same. Since we are interested in the implications of endogenous Schumpeterian growth for business cycles, we prefer to perform detrending with respect to the average technology level.

It can be shown that the average technology level $\overline{A}_t$ is given by

$$\begin{aligned} \overline{A}_t &\equiv \int_0^1 A_t(i)di \\ &= n_{t-1}A_{t-1}^{max} + (1-n_{t-1})n_{t-2}A_{t-2}^{max} + (1-n_{t-1})(1-n_{t-2})n_{t-3}A_{t-3}^{max} + ... \\ &= n_{t-1}A_{t-1}^{max} + (1-n_{t-1})\,\overline{A}_{t-1}\ . \end{aligned} \tag{72}$$

All variables with a trend, including the auxiliary variables, are made stationary, generally requiring the nominal variables to be divided by the product of P_t and some appropriate power of $\overline{A}_t$. Consequently, the detrending of many relevant variables involves the distance of a firm i's technology level relative to the average technology level prevailing in the economy, i.e. $d_t(i) \equiv A_t(i)/\overline{A}_t$.

In particular, we can show that the detrended real marginal cost for firm i is given by

$$mc_t(i) \ = \ d_t(i)^{(\alpha-1)}\,\omega_t\ , \tag{73}$$

where $mc_t(i) \equiv MC_t i/P_t$, and $\omega_t \equiv \Omega/P_t\,\overline{A}_t^{(1-\alpha)}$, while its optimal relative reset price $\phi_t^\star(i) \equiv P_t^*(i)/P_t$ is

$$\phi_t^\star(i) \ = \ d_t(i)^{(\alpha-1)}\,f_t\ . \tag{74}$$

Moreover, detrended real aggregate output amounts to

$$y_t \ = \ \frac{k_t^\alpha\,L_t^{(1-\alpha)}}{\int_0^1 d_t(i)^{(\alpha-1)}di}\ . \tag{75}$$

with detrended real investment in $R\&D$, $x_t \equiv X_t/\overline{A}_t$ equating

$$x_t = \beta\,\frac{n_t}{1+\eta}\,\frac{\lambda_{t+1}}{\lambda_t}\,E_t\,v_{t+1}(A_t^{max})\ , \tag{76}$$

with $\lambda_t \equiv \Lambda_t/(P_t\overline{A}_t)$, $E_t\,v_{t+1} \equiv E_t\,V_{t+1}(A_t^{max})/(P_{t+1}\overline{A}_{t+1})$.

Accordingly, the probability of innovating can be also written as

$$n_t = \left(\frac{x_t}{\zeta\,(\overline{A}_t/A_t^{max})^{-1}}\right)^{1/(1+\eta)}\ . \tag{77}$$

Once, the steady state of the detrended model is computed , the model is loglinearly-approximated around its steady state.

5 Parameter Calibration and Characteristics of the Shocks

5.1 Taking the model to the data

Typically, a calibration exercise consists in restricting the values of the model parameters to target effectively a specified set of stylized facts or moments. Ensuring that the simulated data from the model are directly comparable with the empirical data is a crucial but often, rightly overlooked, step of the process, with not much consequences in an exogenous growth set-up. With Schumpeterian endogenous growth, this is not innocuous as it may well lead to a wrongful comparison.

Traditionally, simulated data around the model steady state are compared with the detrended empirical data. Such an oversight would be benign in the case of an exogenous growth model as growth is deterministic, with no built-in cyclical component. However, with endogenous growth, the growth rate of the economy is affected by cyclical components at medium frequencies. Therefore, a first step that is required is to rebuild the trending simulated data series by adding back the trend, namely

$$ln(Y_t^{trending\,simulated}) = ln(y_t^{cyclical\,simulated}) + ln(A_{t-1}) \tag{78}$$

By constructing a new set of trending simulated variables, we ensure comparability between model and empirical data. An HP filter is applied to both sets of data before the moments and comovements are generated.[11]

5.2 The data

The model's calibration is based on U.S. data. We use Fernald (2017)'s data series on utilization-adjusted total factor productivity (TFP) to build a cumulative TFP series compounded at quarterly rates. Civilian non-institutional population is obtained from the U.S. Bureau of Labour and Statistics to compute the *per capita* variables. The Implicit Price Deflator is used as a proxy of the price level, and all macroeconomic aggregates originate from the U.S. Bureau of Economic Analysis. We use series on Real Gross Domestic Product, Research and Development, Real Gross Private Domestic Investment and Real Personal Consumption Expenditures.

[11] Given issues regarding possible spurious dynamics induced by the Hodrick-Prescott filter, that were raised in particular by Hamilton (2017), statistics using first differences will also be computed in subsequent versions of the paper, as a robustness check.

Parameter	Value	Description
α	1/3	Share of capital in intermediate good production
β	.99	Discount rate
δ	.025	Depreciation rate of physical capital
π	1	Steady-state gross inflation rate
u	1	Steady-state capacity utilization rate
ϵ	6	Elasticity of substitution of intermediate good demand
γ	6	Elasticity of substitution of labour demand
ξ_p	$1-\frac{(1-.66)}{(1-n_{ss})}$	Calvo parameter for prices
ξ_w	.66	Calvo parameter for wages
θ	5	Weight on the disutility of labour
ν	1	Utility parameter that determines the Frisch elasticity of labour supply, $(\frac{1}{\nu})$
h	.5	Consumption habit formation
κ	1	Investment adjustment-cost parameter
σ_{ss}	$\frac{(g_{ss}-1)}{n_{ss}}$	Steady-state value for the extent of knowledge-spillover

Note: As explained further below, the joint determination of n_{ss} and σ_{ss} is made to replicate the long-run value of growth rate of the technological frontier g_{ss}.

Table 1: Key parameters

5.3 Calibration associated with the features akin to that a typical New Keynesian DSGE model

The calibration for most of the parameters pertaining to the New Keynesian aspects of the model is fairly standard. Table 1 presents the values used for calibration of key parameters of the model. The share of capital in the production function α, the discount rate β, the depreciation rate of physical capital δ as well as the monetary policy parameters are set to standard values in the literature. The steady-state gross trend inflation π is set at 1 (i.e. the inflation rate is zero in the steady-state). Moreover, we set both the gross rates of backward wage and price indexation to be equal to one to remove such indexation. We also assume full capacity utilization in steady-state, i.e. $u = 1$. Market power in labour supply and in the provision of intermediate goods leads to, generally agreed upon, wage and price markups of around 20% which is the benchmark in Christiano et al. (2005). This corresponds to an elasticity of substitution of 6 between intermediate goods, as well between labour types. The disutility of labour θ is set so that steady state labour is approximately 0.33 of the hours endowment per period.

Finally, price changes are semi-endogenous in our model as, at any given period, two types of firms are allowed to reset prices, namely the innovative firms that implement a new product and the incumbent ones in operations that have been selected to do so.

5.4 Calibration associated with the Schumpeterian features of the model

Calibrating Schumpeterian models is a relatively new endeavour in economic research. We will base our calibration on the statistical moments of the variables related to research and development and technological advancement.

As shown in Figure 3, from 1960 and 2016, the share of GDP dedicated to $R\&D$ investment has varied between 2.3% ans 2.9%, averaging a 2.56%-share of the GDP over the period. We use this sample average as the steady state target for investment in $R\&D$ by calibrating the production function parameters at $\zeta = 10^8$ and $\eta = 11$.

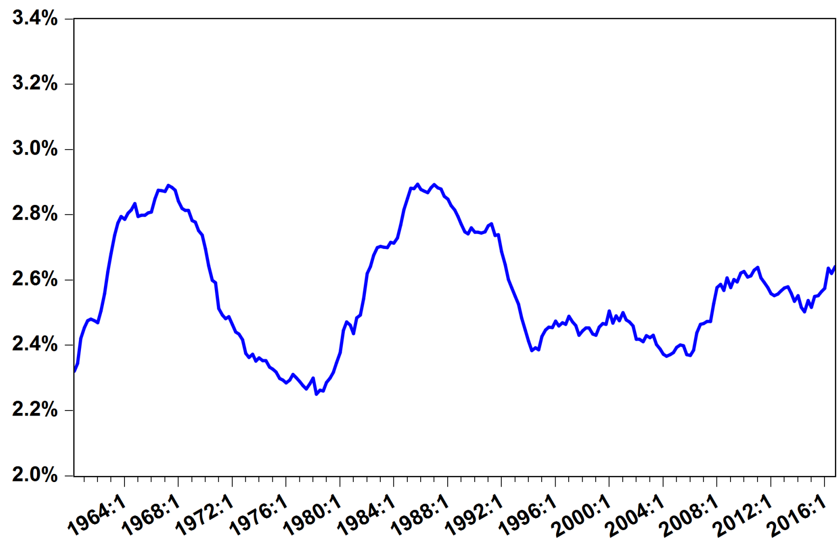

Figure 3: $R\&D$ investment-to-GDP ratio in the United States (1960Q2-2016Q4)

Given the relationship between the frontier growth rate, the spillover and the innovation probability, there are additional degrees of freedom left as the variation along one dimension can be compensated by a change in another. This additional degree of freedom will prove particularly interesting when studying the dynamics of the model. For instance, different restrictions will yield significantly different impulse response functions.

5.5 Persistence and variance of the shocks

Investment and monetary policy shocks parameters are derived from New Keynesian literature and find their sources in Justiniano et al. (2010).

Parameter	Value	Description
ρ_m	0.4	Monetary policy shock persistence
σ_m	0.001	Monetary policy shock variance
ρ_μ	0.2	Investment shock persistence
σ_μ	0.005	Investment shock variance

Table 2: Calibration of the monetary and investment shocks

Parameter	Value	Description
ρ_z	0.906	Technology shock persistence
σ_z	0.0194	Technology shock variance
ρ_σ	0.8	Knowledge-spillover shock persistence
σ_σ	0.002	Knowledge-spillover shock variance

Table 3: Calibration of the technology and knowledge-spillover shocks

The spillover and transitory technology shocks are two components of TFP. The transitory technology shock can be seen as the cyclical component of TFP. We, therefore, calibrate both persistence and volatility to match the cyclical component of TFP. We, then, perform a grid search to minimize the Euclidian distance between the volatilities of observed and simulated TFP.

6 Consistency of the theoretical model with the data

In this paper, the calibration we have adopted was not aiming at replicating with precision the sample statistical moments observed in the data. Yet, comparing some key moment of the HP cyclical components of simulated and empirical data allows for an indirect rough empirical validation of the consistency of the mechanisms included in the model with some interesting relative volatilities and cross-correlations. Given the limited computing power at our disposal, and using a grid search approach, we made (at this time) a compromise between processing time and accuracy.

	Output	Consumption	Investment	*R&D* Investment
Empirical	1	0.7477	4.4758	1.4701
Simulated	1	0.6305	2.7866	1.3001

Table 4: Relative volatilities in the cyclical component of the variables with respect to cyclical output according to the endogenous growth model

As shown in Table 4, the simulated relative volatilities to that of output are consistent with their sample counterparts. In particular, both investment in physical capital and investment in $R\&D$ are more volatile than output respectively with relative volatilities given by 2.78 and 1.30. We notice however that cyclical consumption is somewhat smoother in the model than in the data, and that investment in physical capital is not volatile enough in comparison with the data. By resorting to a smaller increment in the grid-search procedure, and by adjusting the parameters for intensity parameter of habit persistence in consumption as well as the convex adjustment cost of investment in physical capital, it is likely that the moments for the simulated data get even closer to the empirical moments.

The order of magnitude of various cross-correlations between the HP cyclical components of macroeconomic variables presented in Tables 5 and 6 are fairly similar in both empirical and simulated data, albeit they often appear to be a bit too strong for the simulated series from the model. Simulated correlations between $R\&D$ investment and output, and $R\&D$ and consumption are 0.40 and 0.55 compared with 0.36 and 0.31 in the data. It is worth pointing out the model's ability to replicate accurately the low contemporaneous cyclicality of $R\&D$, as well as the contemporaneous cross-correlation between the cyclical components of investment in physical capital and in $R\&D$, that are respectively 0.21 in the model and 0.22 in the data.

	Output	Consumption	Investment
Consumption	0.7805	1	
Investment	0.8503	0.7145	1
$R\&D$ Investment	0.3614	0.3070	0.2207

Table 5: Empirical cross-correlations in the cyclical component of the data

	Output	Consumption	Investment
Consumption	0.9638	1	
Investment	0.9610	0.8541	1
$R\&D$ Investment	0.4065	0.5586	0.2054

Table 6: Cross-correlations in the cyclical component of the simulated data in the endogenous growth model

7 The business cycle implications of varying the steady state probabilities of innovating

In our model, the balanced growth rates of the technological frontier and of the trend growth rate of real output are determined by the product of both the steady-state probability of innovating, n_{ss}, and the steady-state extent of technology spillover, σ_{ss}, as the latter induces a larger technological jump of the frontier. Hence,

$$g_{ss} = 1 + \sigma_{ss} n_{ss} \ . \tag{79}$$

Namely, it is therefore possible to consider different combinations of n_{ss} and σ_{ss}, that generate the same value of g_{ss}. However, existing evidence in the literature does not carry much information about how to calibrate σ_{ss} versus n_{ss}.[12]

Nonetheless, at this stage, it is instructive to consider how the impulse response functions of the macroeconomic variables to each shocks differ when comparing two economies. In the first one, we fix $n_{ss} = 0.05$ and $\sigma_{ss} = 0.097$, while in the second one, these are respectively set as $n_{ss} = 0.35$ and $\sigma_{ss} = 0.0139$. Hence, both economies are consistent with a steady-state quarterly gross growth rate given by $g_{ss} = 1.00485$ or approximately 2% *per annum*. The former economy is therefore associated with a lower a probability of innovating, typically linked with a "bigger" jump, while the latter is characterized by more frequent, yet smaller, discoveries that are more easily spread, so that thet do not require as big a value for σ_{ss}.

Although we keep invariant the steady state real growth rate of output, a different steady-state probability of innovation has a specific impact on the dynamics of the economy, that was absent in the usual model with exogenous growth. Indeed, in our setup, a larger value of n_{ss} implies not only that a greater proportions of intermediate firms will be innovating in the steady state, but also that the extent of effective price flexibility is higher as the proportion of firms that proceed with a reoptimization of the intermediate good prices has risen. That is, since innovating firms are allowed to set the optimal mark-up pertaining to the innovating intermediate product.

We now discuss how the impulse response functions of the cyclical macroeconomic variables following different shocks are altered when considering a "low" steady-state probability of innovation, say 5%, *versus* an economy with a relatively "high" steady-state probability of innovation, say 35%.

[12]Moreover, in the real world, some variety across industries, as well as various era may imply that this decomposition may well differ across time. This would introduce another level of interesting complexity over a long span of data, that is beyond the scope of the current paper.

7.1 The impulse response functions for the cyclical components of the variables following a transitory technology shock

As can be seen in Figures 4 and 5, when changing n_{ss} from 5% to 35%, a positive transitory technology shock does not lead to a very significantly different cyclical dynamics on output, consumption, investment in physical capital, hours worked, capacity utilization, physical capital stock and the optimal reset price. However, this does not mean that it is without consequences. Initially, a cyclical wealth effect dominates on hours worked, while it builds up on consumption. Then, the cyclical intratemporal and intertemporal substitution effects on hours worked takes the lead. Also, as expected the rise in the marginal product of physical capital first leads to a positive cyclical response of capacity utilization, while the increased incentive to invest in physical capital peaks up and start to rise its stock.

With $n_{ss} = 0.35$, there is a sharper immediate negative response of inflation and drop in nominal interest rates lasting a few quarters. Also, the induced rise in the marginal productivities of labour and capital translates into transitory positive cyclical impacts on the wage index, the optimal reset wage and the return on physical capital. Furthermore, this positive transitory positive technology shock has a transitory, albeit persisting beyond 25 quarters on investments into the value of an intermediate firm, $R\&D$ investments, the innovation probability, and finally average real growth.

7.2 The impulse response functions for the cyclical components of the variables following an efficiency of investment shock

After a temporary positive improvement on the efficiency of investment in physical capital, a quick temporary rise in the return on physical capital follows, that quickly brings up temporarily cyclical capacity utilization and private investment. The increase in optimal reset wage and return on capital both push the marginal cost up through the dynamic complementarity of inputs.

The cyclical impulse response functions in Figures 6 are not much different whether $n_{ss} = 0.05$ or $n_{ss} = 0.35$.

As shown in the lower panel of Figure 7, when $n_{ss} = 0.35$, a positive shock on the efficiency of investment leads to a persistent cyclical increase in the probability of innovating, in the discounted expected value of the intermediate firms, and in $R\&D$ investments. Here too, the average growth rate of real output also exhibits a persistent cyclical increase.

7.3 The impulse response functions for the cyclical components of the variables following a monetary policy shock

We now turn to the cyclical impact of a restrictive monetary policy, brought about by a "discretionary" rise in the nominal interest above the Taylor-type reaction function. The net effect on the nominal interest rate therefore is the combination of the monetary shock and the immediate reaction to inflation- and output-growth gaps. As illustrated in Figure 8, the nominal interest rate initially goes up, while the inflation rate goes down. This is accompanied by a quick cyclical downturn in real output, in consumption, in hours worked, in capacity utilization and in investment in physical capital, along with a cyclical decrease in the real return on capital. A higher steady-state innovation-probability value of $n_{ss} = 0.35$, compared with $n_{ss} = 0.05$, generally leads to somewhat weaker cyclical fluctuations in the above variables, except for inflation that is brought down more quickly.

At the same time, there is a smaller negative cyclical impact on marginal cost, while the wage index temporarily goes up, even though the optimal reset wage is cyclically lower.

Meanwhile, this restrictive policy shock causes a fairly more important cyclical downturn in the probability of innovating that lowers a little more the value of intermediate firms and invesments in $R\&D$. However, despite these consequences, the cyclical impact of average real growth rate does not differ as much with either $n_{ss} = 0.35$ or $n_{ss} = 0.05$. The cumulative impact of lower compounded growth is not necessary negligible however. This still needs to be assessed.

7.4 The impulse response functions for the cyclical components of the variables following a knowledge-spillover shock

The last shock to be considered is specific to a model with endogenous growth and has no analogue in exogenous growth business cycle models.

As it is apparent in Figures 10 and 11, the cyclical dynamic responses to a transitory positive knowledge-spillover shock is significantly sensitive to the steady-state probability of innovation. Focussing first our attention to Figure 11, an increased extent of knowledge-spillover cyclically lowers for some quarters both the value of intermediate firms' discounted profits and investments in $R\&D$, while cyclically lowering the probability of innovation. However, the cyclical impacts on the innovation probability and the average real output growth rate are fairly subdued when $n_{ss} = 0.05$. This is not so for $n_{ss} = 0.35$. Instead, the positive cyclical response oft the average growth rate is much higher and more persistent. Moreover, in this case, for about 5 quarters, there initially is a bigger decrease in each of the investment in $R\&D$, the value of the discounted firm and the probability of innovation, then followed by a positive 4- to 5-quarter rise in these variables, prior to a sustained but slight negative impact.

Now considering the cyclical impulse responses of the other macroeconomics variables, an appreciably higher value of n_{ss}, such as 35%, leads to significantly more volatile cyclical effects of a given positive knowledge-spillover shock, as there are, in part, stronger intertemporal substitution effects that are engineered by this shock. To some extent, as seen in Figure 10, it is as if the cyclical impulse response functions are somewhat squeezed for many graphics for a higher value of n_{ss}.

There are two main conclusions we can draw from the impulse response function to the spillover shock. The spillover shock could be the main source of cyclical fluctuation of the growth rate of the economy. An estimated model would allow to compute the variance decomposition and confirm the intuition gathered from the impulse response functions. A larger innovation probability at the steady state quickens the model's reaction to a spillover shock. Most variables such as the nominal interest rate and the optimal reset price return to their steady state much quicker when $n_{ss} = 35\%$. In a future revision, we will also assess the welfare implications of various (σ_{ss}, n_{ss}) pairs. It can be expected that two effects will be in play. On one hand, if a higher value of n_{ss} brings about more volatility of cyclical consumption, this will lower welfare. On the other hand, if it raises trended permanent income, and therefore trend consumption, it will be welfare-raising.

8 The welfare implications of different steady-state probability of innovation

In our model, the innovation probability is the odds that, for a given amount of investments in $R\&D$, the process is successful in introducing a new technology. For the firm in monopolistic position in its sector, it also corresponds to the probability of being supplanted by an innovator. As such, this can be seen as the proportion of firms that enters or exits the unit mass intermediate sector.

Having looked at the impulse response functions of a New Keynesian model with endogenous growth, we now turn to the welfare implications of the interaction between business cycles and endogenous growth. Our aim is to assess the welfare cost of different calibrations of the Schumpeterian growth mechanisms, while keeping constant the steady-state growth rate of the technological frontier. Since $g_{ss} = 1 + \sigma_{ss} n_{ss}$, both n_{ss} and σ_{ss} can be varied while keeping g_{ss} invariant, in order to study the welfare implications of different combinations of steady-state probability of innovation and spillover. For instance, a high degree of knowledge-spillover can compensate for a lower innovation probability.

The infinite-horizon sum of the present discounted value of flow utilities across households define their welfare, which can be recursively expressed as follows:

$$\mathscr{W}_t = U(C_t, L_t) + \beta \mathscr{W}_{t-1} . \qquad (80)$$

Accordingly, applying the methodology developed by Schmitt-Grohe and Uribe (2004), it is possible to make welfare comparisons between a benchmark scenario for the (n_{ss}, σ_{ss})-pair, indexed by subscript B, and alternative pairs, generically indexed by subscript A. We choose to contrast various scenarios relative to a benchmark using consumption-equivalent differences as a more tangible measure of welfare. Namely, it is thus expressed as a percentage of steady state consumption that would make a household indifferent between two scenarios.

A first measure, evaluated at the non-stochastic steady states, reckons the fraction of consumption that would have to be given up each period in the alternative scenario to reach the same welfare level as in the benchmark scenario. It is defined as:

$$\mathscr{C}_{ss} = 1 - exp\Big((1-\beta)\cdot[\mathscr{W}_{B,ss} - \mathscr{W}_{A,ss}]\Big) \tag{81}$$

where $\mathscr{W}_{B,ss}$ is the benchmark and $\mathscr{W}_{A,ss}$ is the alternative. Accordingly, when $\mathscr{W}_{B,ss} < \mathscr{W}_{A,ss}$, then $\mathscr{C}_{ss} > 0$.

A second measure can also be evaluated at stochastic means of the value functions obtained from the model simulations. It hence takes into account the interaction between the alternative (n_{ss}, σ_{ss})-pair interacts with the various random shocks built-in the model. In this case, the corresponding consumption-equivalent welfare loss is given by:

$$\mathscr{C}_{m} = 1 - exp\Big((1-\beta)\cdot\big[E(\mathscr{W}_{B,t}) \; - \; E\big(\mathscr{W}_{A,t}\big)\big]\Big) \tag{82}$$

with $\mathscr{C}_m > 0$ when $E(\mathscr{W}_{B,t}) < E(\mathscr{W}_{A,t})$.

As previously mentioned, different combinations of (n, σ), that yield a yearly steady-state gross growth rate of $g_{ss} = 1.01954$ *per annum*, are used to compute and to compare the non-stochastic steady-state and stochastic-means consumption-equivalent welfare measures. While the former ($\mathscr{C}_{ss}$) is obtained directly, the latter first requires to compute the stochastic means of the variables of interest prior to calculating $\mathscr{C}_m$.

Since $\sigma_{ss} = \frac{g_{ss}-1}{n_{ss}}$, for the calibrated value of $g_{ss} = 1.00485$ per quarter, we set the benchmark calibration at a quarterly steady-state innovation probability of $n_{ss} = 0.15$, along with $\sigma_{ss} = 0.3233$. For the alternative scenarios, the innovation probability n are varied from 0.16 to 0.54. The corresponding spillover parameter σ_{ss} is changed accordingly from 8.9815×10^{-3} to 3.0313×10^{-2}. Figure 12 shows the results from these simulations.

There are two main conclusions. First, the welfare impact of varying the steady-state innovation probability, even for a constant value of g_{ss} is non negligible. Indeed, to be indifferent between an innovation probability given by $n_{ss} = 0.15$ instead of $n_{ss} = 0.23$, a household would have to be compensated with a higher steady state consumption of 2.0% according to the non-stochastic the steady-state measure, and 4.7% according to the stochastic-means measure. Second, there is significant non linearity when comparing the welfare impact of different (n_{ss}, σ_{ss}) environments. Indeed, the consumption-equivalent welfare-comparison curve is positively sloped and steeper between $n_{ss} = 0.16$ and $n_{ss} = 0.2$. It reaches a maximum at $n_{ss} = 0.28$, before slowly decreasing.

Two opposing forces are simultaneously at work in determining welfare. On the one hand, an increase in the steady-state innovation probability leads to a greater turnover in firms who set prices more often. As it is even more strongly revealed by the $\mathscr{C}_m$ measure, a higher value for n_{ss} allows the model economy to be closer to an economy without any price rigidities. Indeed, the reduction in the price wedge is welfare improving. On the other hand, when the innovator decides how much to invest in R&D, since only private benefits are taken into account. Since an increase in the innovation probability amounts also to a lower probability for an incumbent to remain in operation, this reduces the discounted value of the innovation, with its consequential reduction in welfare.

Hence, there is also a positive externality from innovating that is not considered by the private innovator, while the society benefits from additional permanent push of the technological frontier. From a normative angle, The larger the gap between the private and social values of innovating, the more investment in R&D is socially suboptimal. This suggests some issues worth considering in future research on the appropriate length of intellectual property right policy.

9 Conclusion

Despite the development of modern endogenous growth models, commonly used DSGE business cycle models are based on an exogenous neoclassical long-term growth and focus only on the fluctuations around that trend growth. By embedding Schumpeterian innovation into the intermediate production sector within a New Keynesian model with nominal wage and price stickiness, we show that endogenous decisions to invest in *R&D* have implications that impact the likelihood of innovating and pushing the technological frontier, while adding a relevant transmission channel for common shocks (such as aggregate technological, investment efficiency and monetary shocks), as well as for a spillover shock, that was absent in models with exogenous growth.

From a theoretical stance, we show that our hybrid model highlights and addresses new challenges at the modelling and simulation stages. First, the investment decisions in $R\&D$ need to account for the interactions between both the endogenous growth features and the New Keynesian characteristics of the model. Indeed, the expected discounted profits arising from an innovation is affected both by the probability of being supplanted by a new innovator and, if, not supplanted, the probability that the price can be optimally reset in periods subsequent to an innovation. This has implications for the aggregation of the intermediate production sector. Second, the introduction of Schumpeterian innovation, that determines the expected growth rate of the economy, makes it conceivable to consider the implications of knowledge spillover shocks, as an additional dimension not found in the usual business cycle models. This new shock is interpreted as the unpredictable variations and other heterogeneities in the transmission of knowledge and/or abilities to capitalize on new innovations to push the technological frontier further.

From a methodological point of view, it is shown that the computation, presentation and interpretation of the impulse response functions need to be seen differently, when accounting jointly for endogenous growth and business cycles. Namely, the cyclical IRFs of endogenous and exogenous growth models are not directly comparable. The nature of exogenous growth makes it entirely acyclical while endogenous growth rates vary cyclically around a steady state.

Regarding the ability of the model to replicate key stylized business cycle characteristics, adopting a reasonable calibration in line with usual practices, given the arbitrage currently made between accuracy and processing times, we find that the model can produce moments and comovements relating to output, consumption, investment in physical capital and investment in $R\&D$ that are consistent with their empirical counterparts.[13]

Relative to their respective relevant trend, the aggregate business cycles responses to the common shocks defined as disturbances on aggregate productivity, investment efficiency or monetary policy remain quite quantitatively similar in both models with endogenous and exogenous growth, in case where the steady-state probability of innovating in a quarter is fairly small (e.g. 5%). With endogenous Schumpeterian growth, however, a new knowledge-spillover shock, while sensitive to calibration, allows additional insight into the average growth rate of the economy at different frequencies, even beyond those typically considered in the literature. This suggests other dimensions worth pursuing in future research.

[13] In a future paper, the estimation of this model or some of its variant is intended.

Moreover, we have shown that the dynamic responses of macroeconomic variables may differ significantly in amplitude and shape-wise if we vary the steady-state probability of innovating, yet keeping constant the steady-state or trend growth rate of real output. This arises because a higher steady-state probability of innovating, typically linked with a "smaller" jump of the technological frontier, is associated with a smaller steady-state extent of technology spillover, i.e. smaller discoveries are more easily spread.

Finally, we show that there are important welfare implications of different combinations of steady-state innovation probability and extent of knowledge-spillover, even for the same 1.95% *per annum* value for steady-state growth of the technological frontier. In particular, an economy with a 23% steady-state quarterly probability of innovating rather than 15%, accounting for dynamic interactions, has a consumption-equivalent impact on welfare of a 4,7% higher level.

Our model brings to the fore new questions that have not been raised in standard DSGE New Keynesian models and that suggest possible future extensions. How important are the mismeasurements of inflation and of effective real growth due to the growth and business cycle implications of innovation? How should optimal monetary policy be modelled in an environment accounting for innovation? Should the monetary authority's reaction respond differently according to the source of the output and the inflation fluctuations? What are the business cycle and growth implications of fiscal policy in this context, including how should we assess different government policies intended to encourage innovation?

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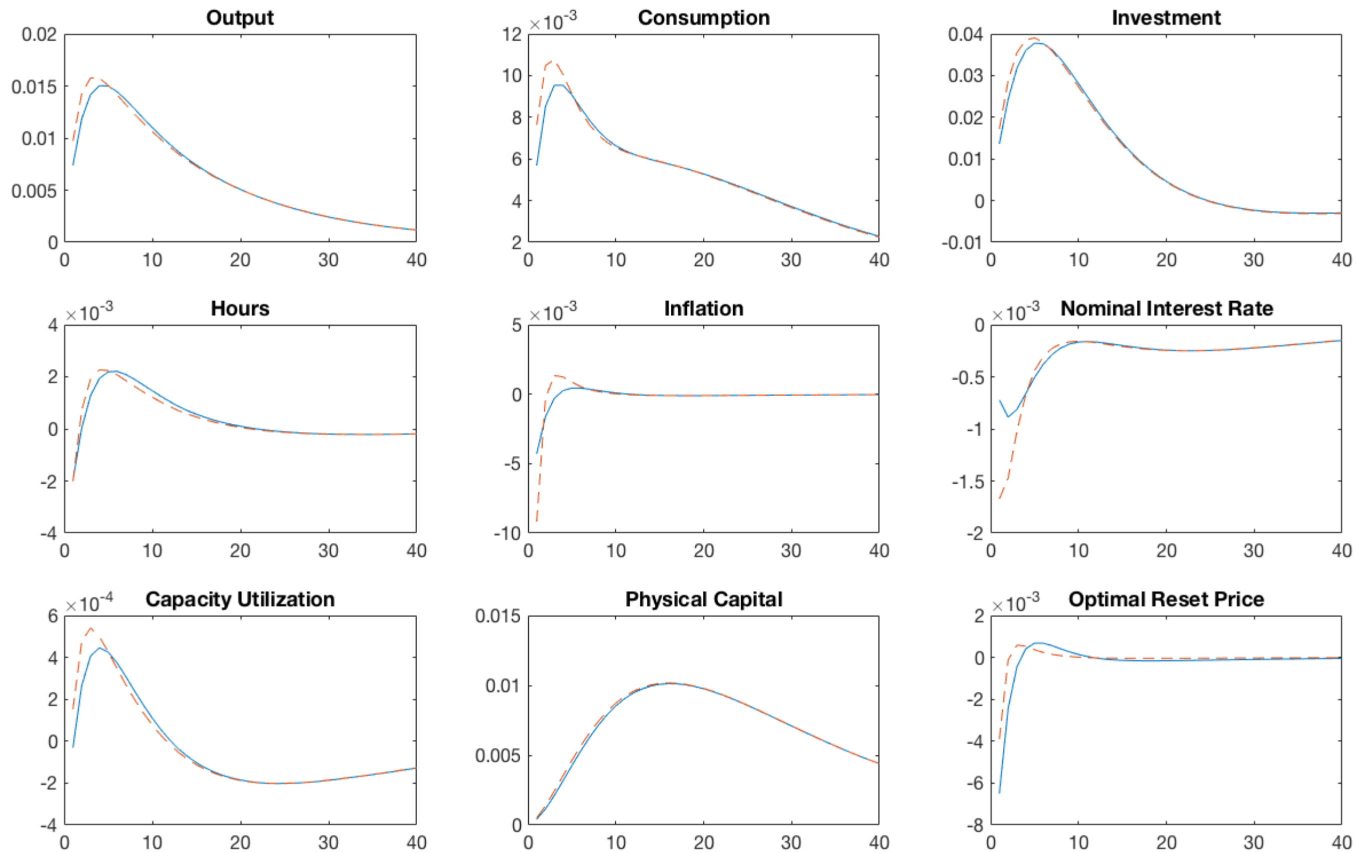

Figure 4: Impulse response functions of the HP cyclical components of macroeconomic variables following a transitory technology shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

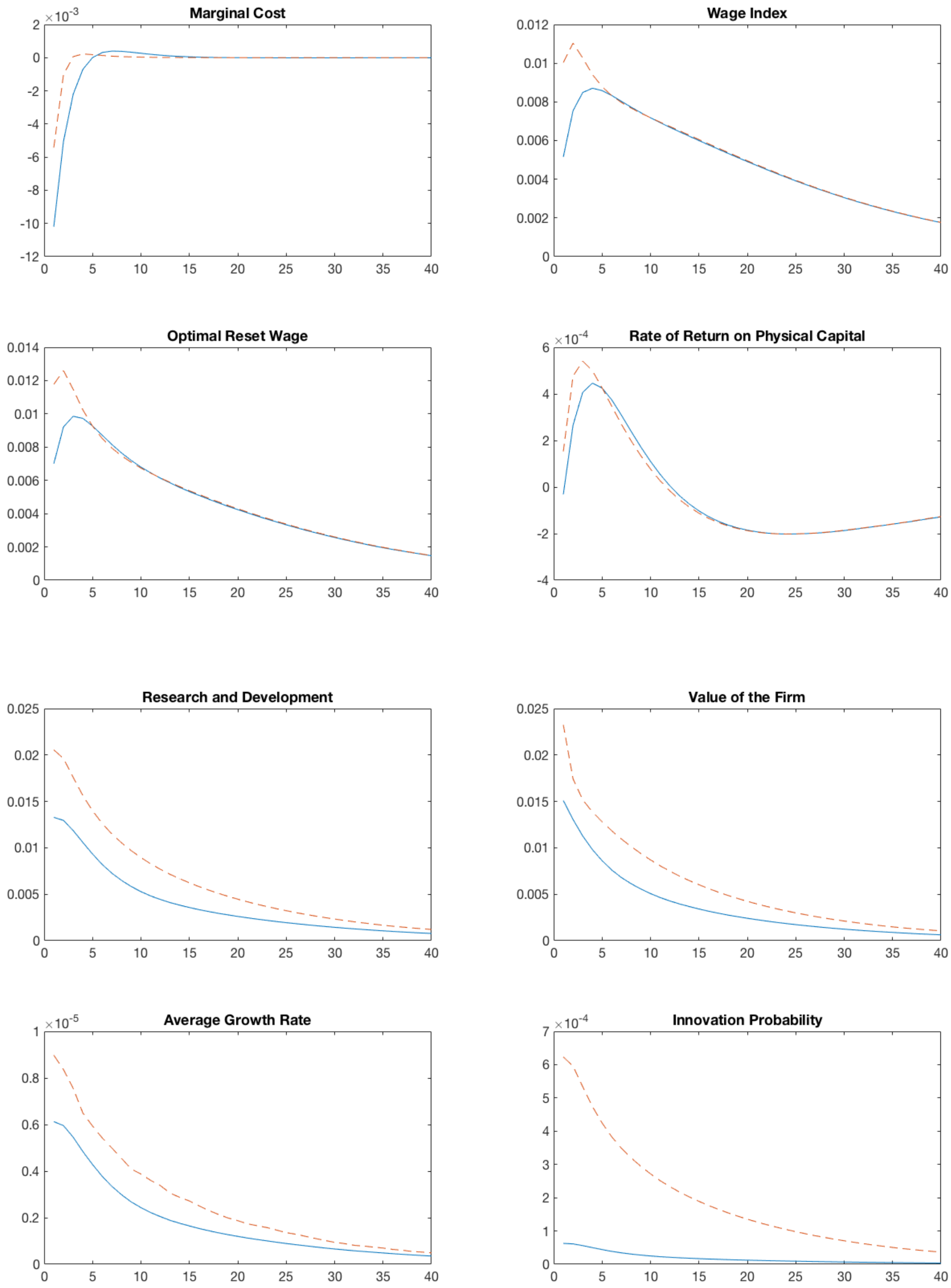

Figure 5: Impulse response functions of the HP cyclical components of macroeconomic variables following a transitory technology shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

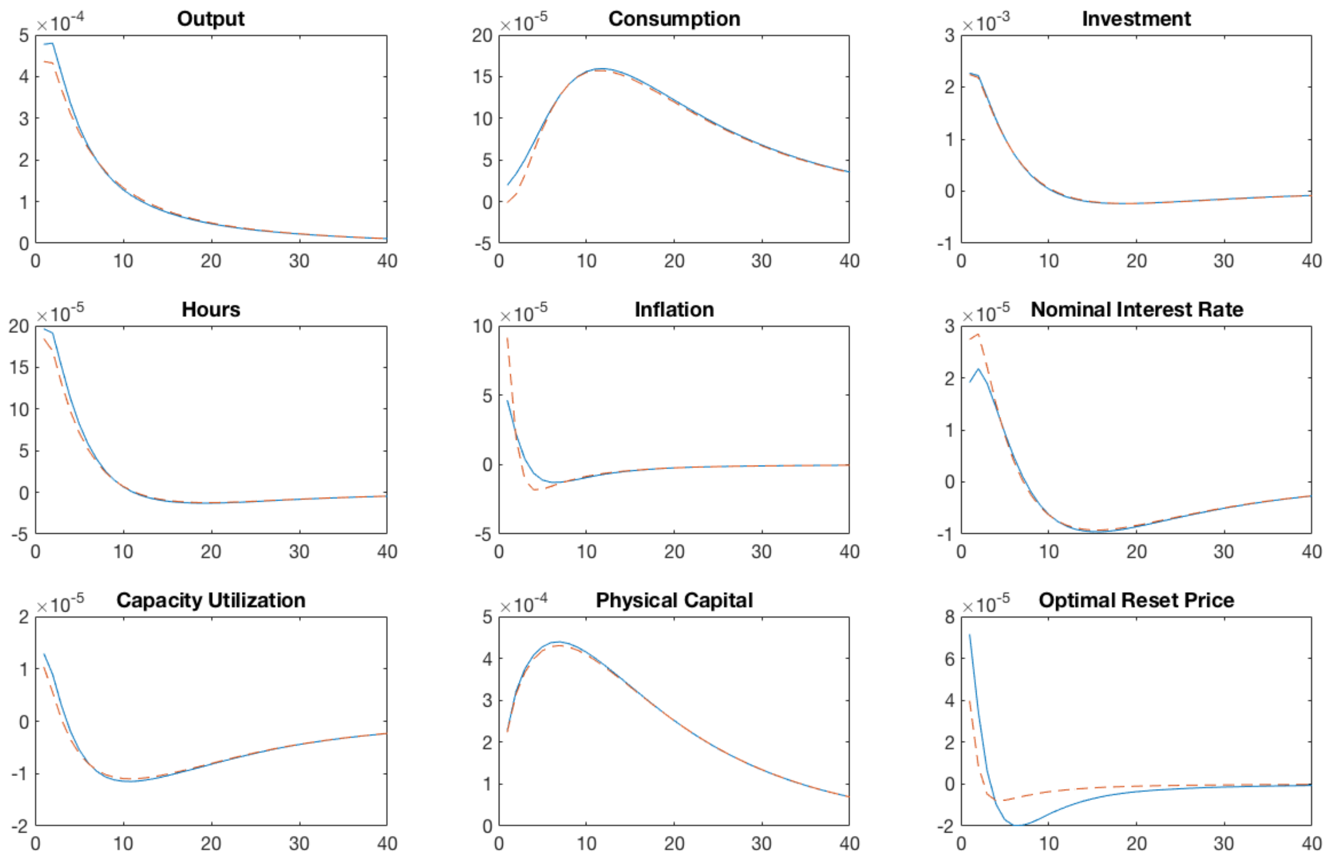

Figure 6: Impulse response functions of the HP cyclical components of macroeconomic variables following an efficiency-investment shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

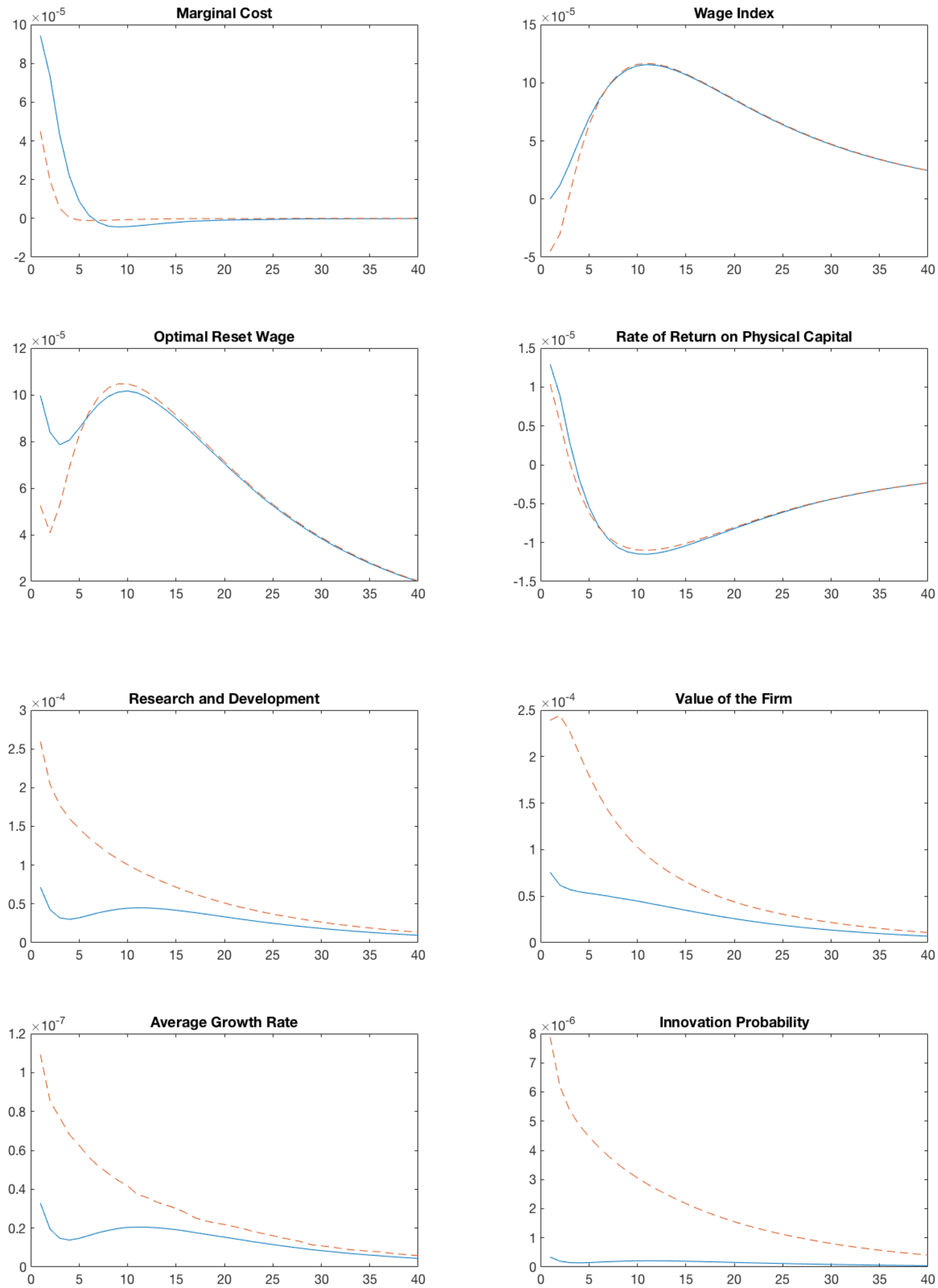

Figure 7: Impulse response functions of the HP cyclical components of macroeconomic variables following an efficiency-investment shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

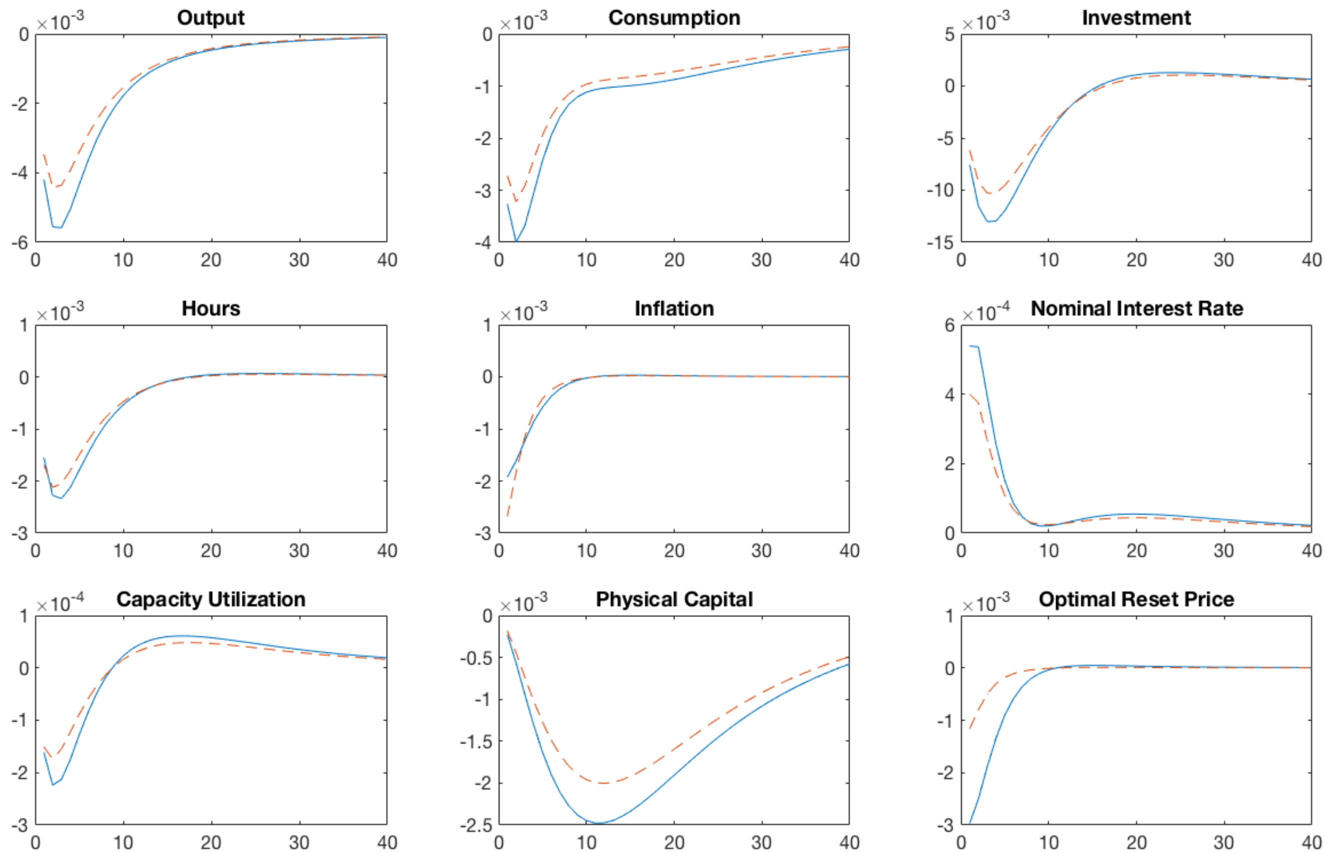

Figure 8: Impulse response functions of the HP cyclical components of macroeconomic variables following a monetary policy shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

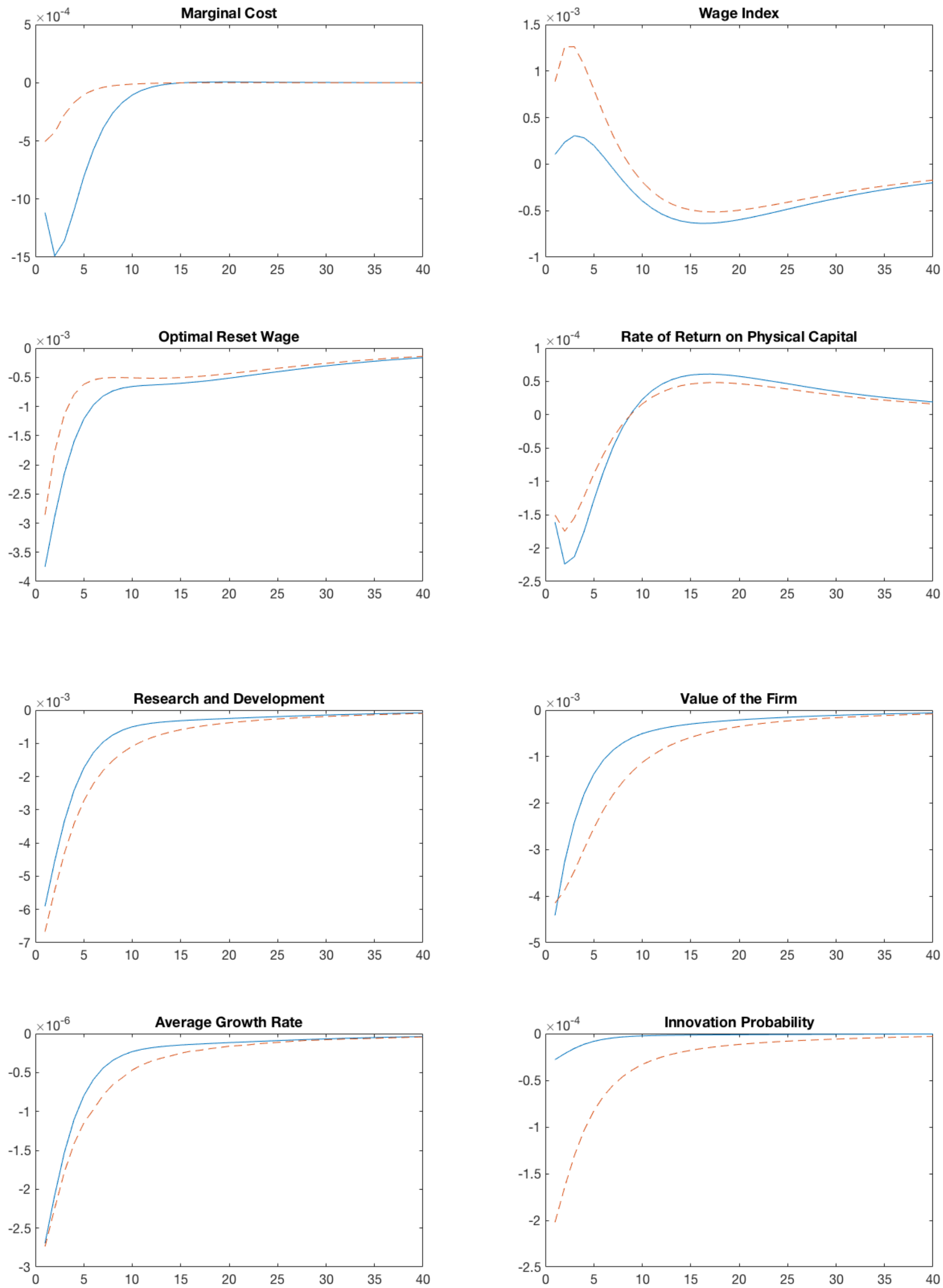

Figure 9: Impulse response functions of the HP cyclical components of macroeconomic variables following a monetary policy shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

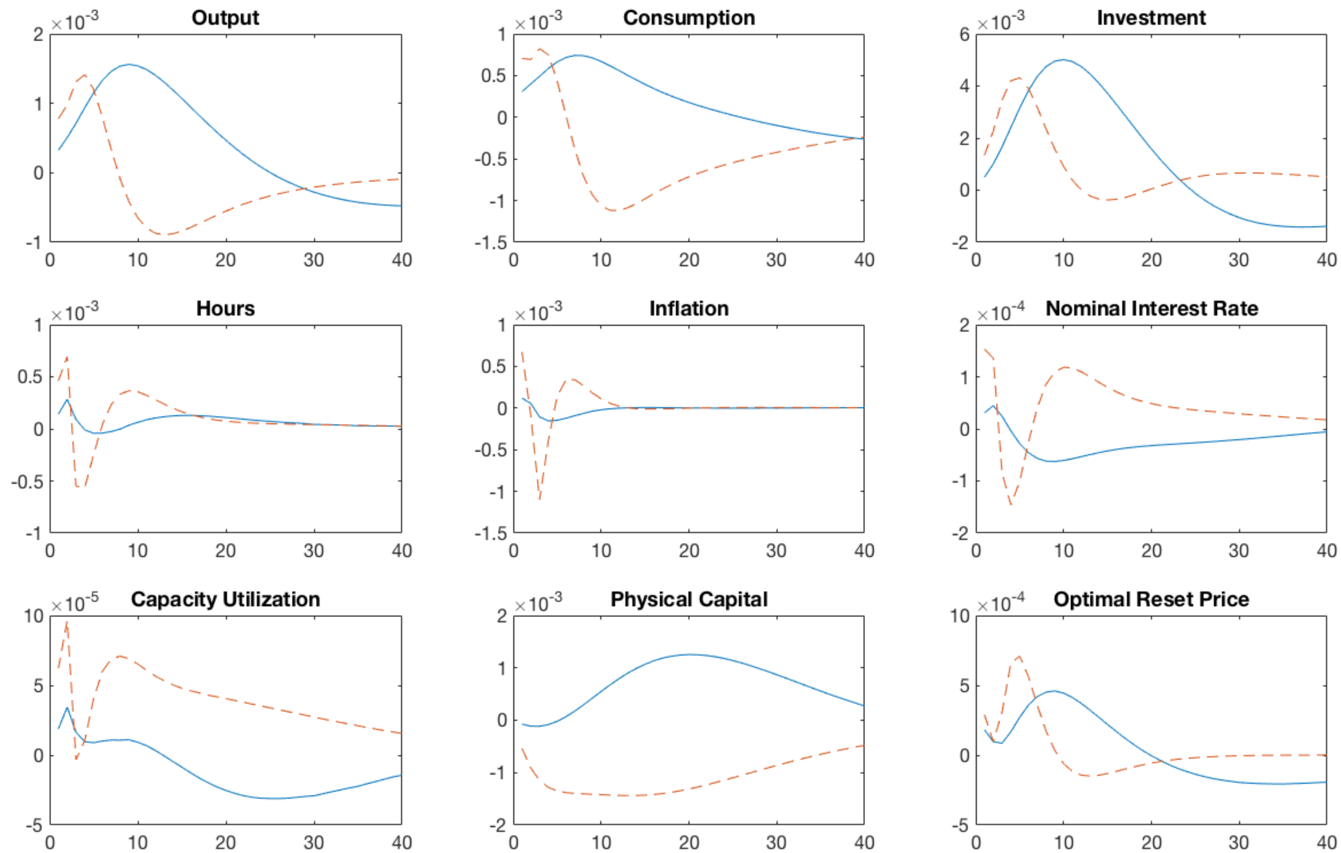

Figure 10: Impulse response functions of the HP cyclical components of macroeconomic variables following a knowledge-spillover shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

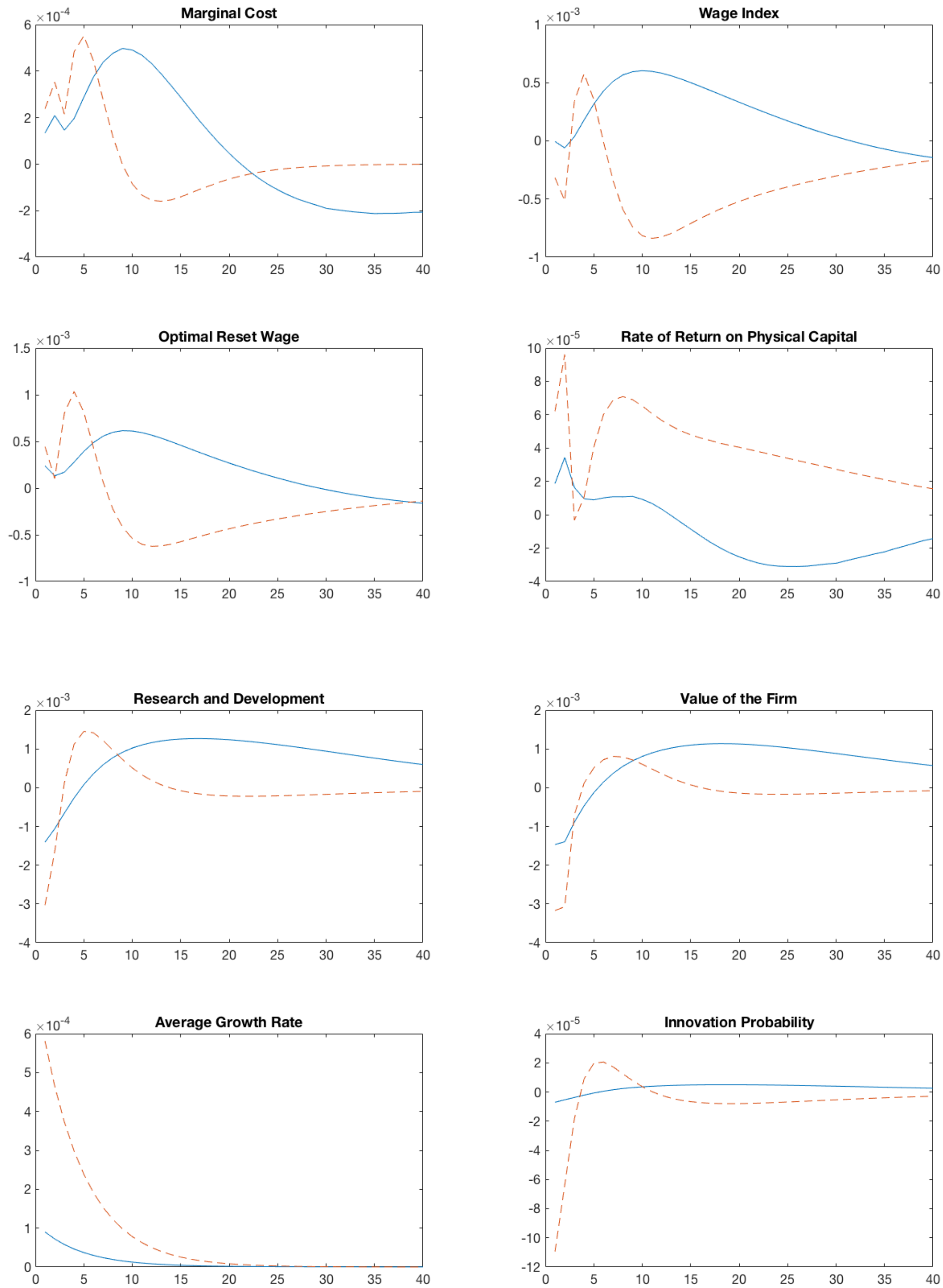

Figure 11: Impulse response functions of the HP cyclical components of macroeconomic variables following a knowledge-spillover shock. (Solid line : $n_{ss} = 5\%$; dashed line: $n_{ss} = 35\%$)

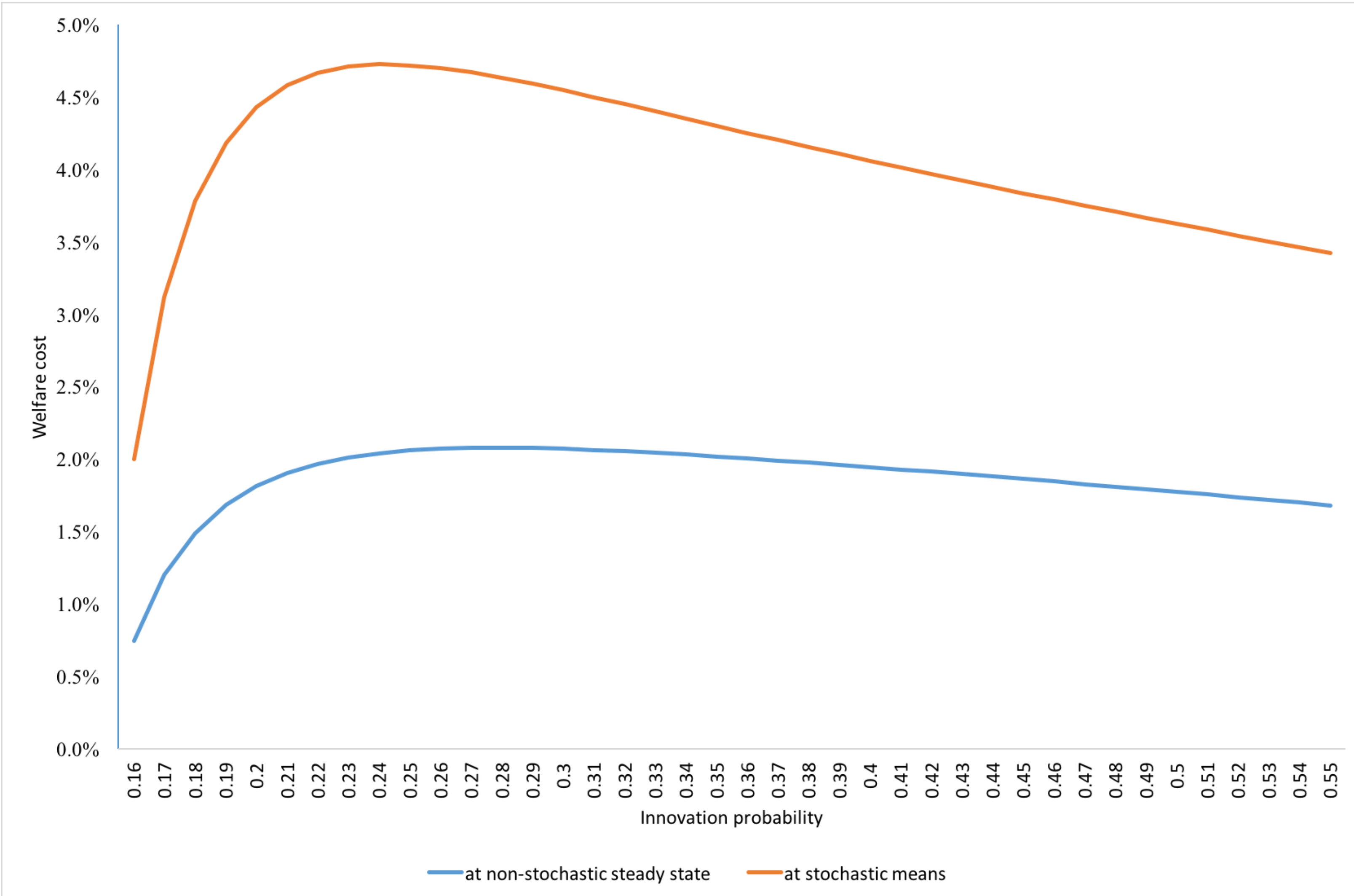

Figure 12: Consumption-equivalent welfare cost of raising the steady-state probability of innovating from $n_{ss} = 0.15$ and beyond, with a given steady-state technological frontier gross growth rate set at $g_{ss} = 1.0195$ per annum or $1.0195^{0.25} = 1.00485$ per quarter.

A The Set of Equilibrium and Optimality Equations

In the equations below, Λ_t and Φ_t are the Lagrange multipliers associated respectively with the household's budget constraint equation (14), and the investment equation (15) at date t.

$$\frac{1}{C_t - hC_{t-1}} - \Lambda_t P_t - \beta \frac{h}{C_{t+1} - hC_t} = 0 \tag{A1}$$

$$-\frac{\Lambda_t}{1 + r_t} + \beta \Lambda_{t+1} = 0 \tag{A2}$$

$$q_t - P_t a'(u_t) = 0 \tag{A3}$$

$$-\Lambda_t P_t + \Phi_t \mu_t \left[\frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) + 1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] + \beta \Phi_{t+1} \mu_{t+1} \frac{I_{t+1}^2}{I_t^2} S'\left(\frac{I_{t+1}}{I_t}\right) \tag{A4}$$

$$-\Phi_t + \beta \Lambda_{t+1} \big(q_{t+1} u_{t+1} - P_{t+1} a(u_{t+1})\big) + \beta \Phi_{t+1} (1 - \delta) = 0 \tag{A5}$$

$$K_t = u_t \widetilde{K}_t \ . \tag{A6}$$

$$W_t^*(j)^{-\gamma\nu - 1} = \frac{\gamma - 1}{\theta\gamma} \frac{Aux_{bc,t}}{Aux_{dis,t}} \tag{A7}$$

$$\frac{K_t}{L_t} = \frac{\alpha}{1 - \alpha} \frac{W_t}{q_t} \tag{A8}$$

$$F_t = \frac{\epsilon}{\epsilon - 1} \frac{Aux_{cost,t}}{Aux_{rev,t}} \tag{A9}$$

$$n_t = \left(\frac{X_t}{\zeta\, A_t^{max}} \right)^{1/(1+\eta)} \tag{A10}$$

$$A_t^{max} \ = \ g_t^{max} \bullet A_{t-1}^{max} \tag{A11}$$

$$g_t^{max} \ = \ 1 \ + \ \sigma_t \, n_{t-1} \tag{A12}$$

$$X_t = \beta \, \frac{n_t}{1 + \eta} \, \frac{\Lambda_{t+1}}{\Lambda_t} \, \frac{E_t V_{t+1}(A_t^{max})}{P_t} \ . \tag{A13}$$

$$E_t \, V_{t+1} = A_t^{max\,(\alpha-1)(1-\epsilon)} \bullet \big(F_{t+1}^{1-\epsilon} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1} \ + \ Aux_{rem,t+2}\big) \tag{A14}$$

$$P_t^{1-\epsilon} \;=\; \xi_p (1-n_{t-1})(P_{t-1}\pi_{p\,t-1,t})^{1-\epsilon} \\ +\;(1-\xi_p)(1-n_{t-1})F_t^{1-\epsilon}\,Aux_{oldtech,t-1}\;+\;n_{t-1}A_{t-1}^{max\,(\alpha-1)(\epsilon-1)}F_t^{1-\epsilon} \tag{A15}$$

$$W_t^{1-\gamma} \;=\; \xi_w\,(W_{t-1}\,\pi_{w\,t-1,t})^{1-\gamma} + (1-\xi_w)\,W_t^{*\,1-\gamma} \tag{A16}$$

$$P_t^{\epsilon}\,Y_t \;=\; \mu_{Z,t}\,\frac{K_t^{\alpha}L_t^{1-\alpha}}{Aux_{output,t}} \tag{A17}$$

$$C_t \;+\; I_t \;+\; a(u_t)\widetilde{K}_t \;+\; X_t \;=\; Y_t\;, \tag{A18}$$

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{Y_{t-1}}\,g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R}\;\mu_{M,t} \tag{A19}$$

Auxiliary variables:

$$Aux_{bc,t} = \Lambda_t\,W_t^{\gamma}L_t + \xi_p\beta(\pi_{w\,t,t+1})^{1-\gamma}Aux_{bc,t+1}\;, \tag{A20}$$

$$Aux_{dis,t} = \Lambda_t W_t^{\gamma(1+\nu)}L_t^{1+\nu} + \xi_p\beta(\pi_{w\,t,t+1})^{-\gamma(1+\nu)}Aux_{dis,t+1}\;. \tag{A21}$$

$$Aux_{rev,t} = P_t^{\epsilon}Y_t + \xi_p\beta\frac{\Lambda_{t+1}}{\Lambda_t}(1-n_t)(\pi_{p\,t,t+1})^{1-\epsilon}Aux_{rev,t+1}\;, \tag{A22}$$

$$Aux_{cost,t} = \Omega_t P_t^{\epsilon}Y_t + \xi_p\beta\frac{\Lambda_{t+1}}{\Lambda_t}(1-n_t)(\pi_{p\,t,t+1})^{-\epsilon}Aux_{cost,t+1}\;. \tag{A23}$$

$$Aux_{rem,t+2} = (1-\xi_p)\beta^2\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\left(F_{t+2}^{(1-\epsilon)}Aux_{rev,t+2} - F_{t+2}^{-\epsilon}Aux_{cost,t+2}\right) \tag{A24}$$

$$Aux_{oldtech,t-1} \;=\; n_{t-2}\,A_{t-2}^{max\,(\alpha-1)(1-\epsilon)} \;+\; (1-n_{t-2})\,Aux_{oldtech,t-2} \tag{A25}$$

$$Aux_{output,t} \;=\; \xi_p(1-n_{t-1})(P_{t-1}\pi_{p\,t-1,t})^{-\epsilon}Aux_{oldtechnonreset,t-1} \\ +\,(1-\xi_p)(1-n_{t-1})F_t^{-\epsilon}Aux_{oldtechreset,t-1} \\ +\,n_{t-1}F_t^{-\epsilon}(A_{t-1}^{max})^{(1-\epsilon)(\alpha-1)} \tag{A26}$$

$$Aux_{oldtechnonreset,t} \;=\; n_{t-1}A_{t-1}^{max\,\alpha-1} + (1-n_{t-1})Aux_{oldtechnonreset,t-1} \tag{A27}$$

Laws of motions for the various stochastic shocks:

$$\ln \mu_{I,t+1} = \rho_I \ \ln \mu_{I,t} + \epsilon_{I,t} \tag{A28}$$

$$\ln \mu_{Z,t} = \rho_Z \ \ln \mu_{Z,t-1} + \epsilon_{Z,t} \tag{A29}$$

$$\ln \sigma_t = \ln \sigma \ + \ \rho_\sigma \ \ln \sigma_{t-1} \ + \ \epsilon_{\sigma,t} \tag{A30}$$

$$\ln \mu_{M,t} = \rho_M \ \ln \mu_{M,t-1} + \epsilon_{M,t} \tag{A31}$$