Pension System and Fiscal Policy in an Economy with Heterogeneous Agents[1]

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Abstract

Fiscal distress worsened by demographic trends intensifies the urgency of carrying out pension reforms in order to insure the sustainability of social security systems in the future. We determine an optimal combination of fiscal instruments and compare social welfare in the case of balanced and unbalanced pension system. The analysis is based on the overlapping generations model with an endogenous interest rate and the pension fund run with deficit, covered by the government, in an economy with two types of agents: Ricardian and non-Ricardian. When all the agents are Ricardian it is optimal to finance the deficit only by an income tax. In the case of an unbalanced pension system, the optimal fiscal policy includes a corner solution with zero social contributions and positive income tax. In heterogeneous economy after a certain threshold level of the share of non-Ricardian consumers, non-zero social contributions are optimal with the lower level of income tax. This result remains the same when levels of social contributions differ for Ricardian and non-Ricardian agents: with an increase of the share of non-Ricardian agents their social contributions become positive, with zero social contributions of Ricardian agents. The optimal choice of government expenditure is increasing with the share of non-Ricardian consumers, compensating the decrease of their welfare at retirement.

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Introduction

Most OECD countries have conducted pension reforms recently in order to ensure both financial sustainability of the pension systems and adequacy of pensions. This underlines the need for an analysis of the consequences of pension reforms and future public expenditure on pension provision.

Urgency of these reforms has been dictated by: first, the challenges to financial sustainability of the existing pension systems, putting additional pressure on public finances (which worsened after the financial crisis of 2008–2009 and sovereign debt crisis); and second, by the longevity coupled with sharply decreasing birth rates. Despite undertaking pension reforms, public expenditure on pensions as a percentage of GDP is expected to rise further in the near future in most OECD countries. According to OECD estimates, public spending on pensions has increased from 6.7% of GDP in 2000 to 8.2% of GDP in 2013, being the largest part of social expenditure (18% on average of the total expenditures in 2013).[3] Long-term OECD forecasts for public spending on pensions suggest that this tendency would continue for most OECD countries, with the spending growing in 21 countries and falling in 14. As a result, pension expenditures are expected to increase from 8.9% of GDP in 2013–2015 to 9.5% of GDP by 2050, reaching 10.9% in 2060.[4]

The change of pension system characteristics such as higher retirement age or the rate of social contributions may be considered as an alternative to standard consolidation measures of public finance. Public pension systems also have a redistributive role, providing the minimum level of income at the retirement for low income households who were not able to save during their working period (minimum pensions are present in 15 OECD countries).

At the same time, an account for the existence of non-Ricardian agents (Galí et al., 2007), who consume all their disposable income and do not make any savings, is crucial in the analysis of fiscal policy (e.g. Almeida et al., 2013). It allows investigating more carefully the consequences of optimal fiscal policy and an economic impact it may have. The structure of the population should be taken into account, since fiscal policy directly affects the consumption of non-Ricardian agents and, as a result, the other macroeconomic variables.

Introduction

A wide range of pension reforms has been launched in many countries in order to ensure the sustainability of the social security system, which is at risk because of increasing expenditures on pensions. The main drivers of these dynamics are demographic changes: fertility rates which are below the replacement level and increasing life expectancy. These demographic developments are leading to a significant increase in the old-age dependency ratio: the ratio is expected to rise from 29.9 in 2016 to 52.2 in 2070 in the EU-27, while the very old-age dependency ratio will increase from 8.4 to 23.0 during this period, according to recent forecasts from the European Commission.[5] Despite the implemented reforms, federal

[3] Pensions at a Glance (2017), OECD Social Expenditures Database (SOCX).
[4] The numbers for 2013–2015 may differ from the values presented in OECD Social Expenditures Database (SOCX) because of the different range of benefits covered and the definitions used (Pensions at a Glance, 2017).
[5] European Commission, The 2018 Ageing Report: Underlying Assumptions and Projection Methodologies, Nov. 2017. Very old-age dependency ratio refers to 80+/15-64.

transfers remain one of the key sources to balance the budget for pension funds. These transfers are expected to rise steadily in the future: this part of fiscal expenditures will increase in OECD countries from 9.3% of GDP in 2010 to 11.7% of GDP in 2050.[6] In Russia, public expenditure on the pension provision is projected to increase from 6.7% of GDP in 2017 to 7.5% of GDP by 2025.

Pension reforms can be considered as an alternative to the traditional measures of fiscal consolidation. This study defines the optimal fiscal instruments (rate of social contributions, income tax, government expenditure) chosen by the social planner under an unbalanced pension system in the economy with Ricardian and non-Ricardian agents (rule-of-thumb consumers),[7] who consume all their disposable income each period. First, we specify how optimal social contributions and income tax change with the retirement age, life expectancy, and labor productivity, and how they depend on the type of pension system (balanced or unbalanced) in the absence of non-Ricardian agents. Second, we define the choice of social contributions, income tax and government expenditure by a benevolent government in an economy with both Ricardian and non-Ricardian agents. The focus is on how optimal policy instruments change with the share of non-Ricardian consumers.

Models with non-Ricardian agents have become common in the analysis of fiscal policy. The inclusion of population heterogeneity via Ricardian and non-Ricardian consumers allows us to capture the failure of Ricardian equivalence.[8] There are several papers that analyze structural fiscal reforms and demographic shocks within the Global Integrated Monetary and Fiscal model (GIMF)[9], which incorporates both Ricardian and non-Ricardian consumers (Almeida et al. 2013a, b; Castro et al., 2015;[10] Karam et al., 2010[11]). However, as opposed to the current research they do not consider the optimal choice of instruments by a public planner and the impact of the share of non-Ricardian consumers in this choice.

The research on pension reforms can be classified by the type of pension system under consideration. The first group consists of research on PAYG reforms (e.g. Nickel et al., 2008; Karam et al., 2010; Kilponen et al., 2006[12]; Castro et. al., 2017; Almeida et al., 2013 a, b; Pierrard, Snessens, 2009; Marchiori, Pierrard, 2012, 2015[13]). This research falls into the first category. Other research considers the switch from pay-as-you-go (PAYG) to a fully funded

[6] OECD, Pensions at glance 2013.

[7] See Galí et al. (2007).

[8] The impact of fiscal policy shocks on private consumption can be captured by the introduction of heterogeneity in consumers (Galí et al., 2007).

[9] The baseline structure of model is based on GIMF, presented in Kumhof, Laxton (2007, 2009, 2013), Kumhof et al. (2009, 2010).

[10] Almeida et al. (2013a) extend the Portuguese Economy Structural Small Open Analytical model incorporating financial frictions (as in Bernanke et al., 1999). The model is applied in research on the assessment of structural reforms, fiscal stimulus and consolidation (Almeida, Castro, Felix, 2009, 2010; Almeida et al., 2010a, b).

[11] Karam et al. (2010) consider three reforms of the PAYG pension system: an increase in the retirement age, cuts in pensions and an increase in social contributions. They show that pension reforms can have a positive effect both in the short run, due to an increase in consumption, and in the long run, due to the lower level of public debt.

[12] Kilponen et al. (2006) extend the OLG model of Gertler (1999) introducing distortionary taxation, time-varying retirement and death probabilities to the model. The model is used for the analysis of demographic changes in a small open economy, using the example of Finland.

[13] Pierrard and Snessens (2009) and a series of further research studies (Marchiori and Pierrard 2012; 2015) sequentially extend the OLG model of Auerbach and Kotlikoff (1987). The model is calibrated for Luxembourg (Luxembourg Overlapping generation model for policy analysis).

pension system (Borsch-Supan et al., 2006; McGrattan, Prescott, 2017). Both types of pension systems were analyzed in Marchiori et al. (2011) and de la Croix et al. (2013). There are also several studies (Galasso, 2008;[14] Heijdra, Romp, 2009[15]; Beetsma et al., 2013) which consider an optimal pension plan or optimal retirement age.

The analysis is based on the overlapping generations model (OLG) initially developed by Yaari (1965) and Blanchard (1985) and extended further by Buiter (1988), Giovannini (1988), Weil (1989) and Bovenberg (1993). In order to investigate the optimal policy mix of social contributions and income tax rate, we extend the model of Heijdra and Bettendorf (2006). They analyzed the economic consequences of lower pensions and a higher retirement age in an open economy with traded and non-traded sectors (as in Turnovsky, 1997).[16] However, Heijdra and Bettendorf (2006) consider an exogenous interest rate along with a rudimentary balanced pay-as-you-go (PAYG) pension system.[17] While Heijdra and Bettendorf (2006) consider the consequences of shocks to the welfare of each generation, we investigate the optimal set of measures conducted by a benevolent government, which maximizes the social welfare function, and how this set of measures is affected by the share of non-Ricardian consumers. Their model is extended to investigate the unbalanced budget of the pension fund in a closed economy with an endogenous interest rate. This allows us to account for the effect of different economic characteristics (retirement age, life expectancy, labor productivity) on capital accumulation. The model is extended to incorporate both Ricardian and non-Ricardian consumers in order to investigate how the share of non-Ricardian consumers affects the choice of policy instruments. Following Heijdra and Bettendorf (2006) the model also incorporates decreasing-with-age labor productivity which adds non-Ricardian characteristics to the model.

Heijdra and Bettendorf (2006) consider two types of demographic shocks: a proportional decrease of both birth and death rates so that population growth remains constant, and a fall of the birth rate accompanied by a growth of the death rate, which leads to a decreasing population size. They consider two types of pension reforms: a decrease in pensions and an increase in the retirement age. Both reforms are followed by an adjustment in social contributions because Heijdra and Bettendorf (2006) consider a balanced pension system. If the interest rate exceeds population growth, both reforms lead to an increase in consumption and financial assets in the long-run, yet also to a decrease in capital, because there is a higher consumption of non-tradable goods. They also analyze the consequences of reforms on the welfare of different generations. A decrease in pensions leads to the standard result of a fall in the welfare of the oldest persons living in the period of the shock.[18] An

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[14] Galasso (2008) within a calibrated model of Galasso and Profeta (2004) extended it to incorporate an endogenous retirement age by conducting simulations up to 2050 for France, Italy, the United Kingdom and the USA, showing that the retirement age and social contributions rate would be increased in all countries considered except Italy.

[15] Heijdra and Romp (2009) analyzed the households' retirement age decisions in the model with exogenous interest rate and endogenous labor supply (developed by Heijdra and Romp, 2006; 2008). They show that for pension reform to have an impact on the households' decisions, the change needs to be significant.

[16] Studies on population aging usually employ dynamic calibrated computable general equilibrium (CGE) models as Auerbach and Kotlikoff (1987).

[17] Pension system is similar to Nielsen (1994).

[18] This result is similar to that from the standard two-generation Diamond (1965) model.

increase in the retirement age does not affect the welfare of retirees and increases the welfare of future generations. The burden of reforms in this case falls on the working generation.

The framework of Heijdra and Bettendorf (2006) and Nielsen (1994) was extended by Nickel et al. (2008), who consider an unbalanced pension system.[19] They analyze three fiscal scenarios in an economy with a decreasing population (Buiter, 1988): a suspension of the public pension system and a decrease in lump-sum labor tax; the suspension of the public pension system and a decrease in distortionary corporate tax; and an increase in the retirement age. Their results suggest that the adverse consequences of pension reforms can be decreased by appropriate taxation policies. If the tax reform is conducted promptly, the negative effect of reform on consumption could be offset, while the public debt would reach a lower equilibrium level.[20]

While Nickel et al. (2008) consider the government as a non-maximizing entity and investigate how the predetermined changes in policy instruments would affect the transition of the main macroeconomic variables to the new equilibrium in an open economy, we define a socially optimal fiscal policy (social contributions, government expenditure and income tax) and compare the set of policy instruments in equilibrium with both an increasing and decreasing population in a closed economy with an endogenous interest rate and a heterogeneous population.

The fact that the income tax rate and social contributions are substitutes is illustrated by the comparison of the optimal policy mixes under both balanced and unbalanced pension systems. Under a balanced pension system social contributions are strictly positive and decrease with population growth, while income tax rate is constant and does not depend on it. In the case of an unbalanced pension system, on the contrary, the corner solution is optimal: social contributions are at zero while the income tax rate decreases with population growth. This policy mix remains optimal in the steady state regardless of life expectancy and labor productivity. Under an unbalanced pension system the optimal income tax rate changes with the structural characteristics of the economy while social contributions are at zero.

Within the model which incorporates Ricardian and non-Ricardian consumers we show that there exists a threshold level of the share of non-Ricardian consumers after which higher social contributions and lower income tax are optimal. It is shown that the optimal share of government expenditure is increasing with the share of non-Ricardian consumers compensating their welfare at retirement.

The paper is organized as follows. Sections 2 present an extended OLG model of Heijdra and Bettendorf (2006) with an unbalanced pension system in a closed economy with an endogenous interest rate. Section 3 presents the comparison of the optimal policy mix and social welfare under balanced and unbalanced pension systems in an economy populated by Ricardian consumers. It is also shown how the optimal policy mix depends on retirement age, life expectancy and labor productivity. Section 4 provides the results for the optimal fiscal policy in the economy with Ricardian and non-Ricardian consumers, with and without a

[19] They also extend the model by assuming that firms issue equities and face adjustment costs in investment.

[20] It was shown that a decrease of pensions to zero, de-facto abolishment of the pension system, is accompanied by the decrease of the distortionary tax on the firms' output, the negative effect on the consumption per capita can be lowered in the short-run, while in the long-run the output and consumption are higher with lower levels of private and public debt.

pension system. It also includes an analysis of the optimal fiscal policy when discrimination among agents based on their type is possible (the case of type specific social contributions is considered). Section 5 concludes the study.

2. The model with Ricardian and non-Ricardian consumers

The model of Heijdra and Bettendorf (2006) is extended by introducing an unbalanced pension system with the deficit covered by a benevolent government, which conducts fiscal policy to maximize social welfare. We consider a closed economy with an endogenous interest rate.

The economy consists of two types of households, firms, the government and the pension fund. Infinitely-lived Ricardian households maximize the present value of utility from consumption taking into account life expectancy. Non-Ricardian consumers, whose share is λ, consume all their disposable income, do not save, and do not have the ability to smooth consumption over time. Both types of consumers work and pay income tax throughout their lives, pay social contributions until the retirement age and receive pensions at retirement. Pensions are paid by the pension fund, the deficit of which is covered by the government. Public debt is financed by bonds held by the households and income tax payments.

In the model upper case variables are aggregates, lower case variables with a bar denote individual variables, and lower case variables without any notation are aggregates per efficiency units of labor.

2.1. Ricardian and non-Ricardian households

The representative **Ricardian type consumer** born at time υ maximizes at each moment t the expected present value of instantaneous lifetime utility of consumption:

$$U^R(\upsilon,t)=\int_t^{\infty}\left[(1-\kappa)\ln\bar{c}^R(\upsilon,\tau)+\kappa\ln g\right]e^{(\rho+\beta)(t-\tau)}d\tau, \tag{1}$$

where $\bar{c}^R$ is personal consumption of Ricardian consumers, g is public spending and $\kappa>0$ is its share in the utility function, $\rho>0$ is the rate of time preference and $\beta\geq 0$ is the probability of death.

Following Bettendorf and Heijdra (2006) we consider a pay-as-you-go (PAYG) pension system introduced by Nielsen (1994). The households pay income taxes throughout their lives, pay social contributions $\bar{t}_W$ until the retirement age of π and receive pensions $\bar{z}$ at retirement. The threshold level π can only loosely be considered as a retirement age because the households continue to work after it.

Ricardian consumers can save part of their disposable income, which can be invested in capital goods, $\bar{k}$, and government bonds, $(\bar{a}^G)$. They pay income tax on their labor income and receive an interest rate $r(t)$ on their financial wealth, $\bar{a}(\upsilon,t)$. The payment $\beta a(\upsilon,t)$ is the actuarially fair annuity paid by the life insurance company.[21] Interest and non-interest net labor income, $WI(\upsilon,t)$, are spent on consumption and saving. Household financial wealth, therefore, consists of capital goods, $\bar{k}$, and government bonds, $(\bar{a}^G)$, both denominated in terms of consumer goods.

The budget constraint of a Ricardian type consumer in terms of the consumption good is:[22]

$$\dot{\bar{a}}(\upsilon,t) = (r(t)+\beta)\bar{a}(\upsilon,t) + WI^R(\upsilon,t) - \bar{c}^R(\upsilon,t) \tag{2}$$

$$\bar{a}(\upsilon,t) = \bar{k}(\upsilon,t) + \bar{a}^G(\upsilon,t), \tag{3}$$

$$WI^R(\upsilon,t) = \begin{cases} (1-t_L)W^N(\upsilon,t) - \bar{t}_W & \text{for } t-\upsilon \le \pi, \\ (1-t_L)W^N(\upsilon,t) + \bar{z} & \text{for } t-\upsilon > \pi, \end{cases} \tag{4}$$

where $W^N(\upsilon,t)$ is the wage at time t of the worker born at time υ.

Labor supply for both types of consumers is non-elastic: each household supplies one unit of labor.[23] Labor productivity decreases with the age of the worker. The worker of generation υ at time t supplies $n(\upsilon,t)$ efficiency units of labor (Bettendorf and Heijdra, 2006):

$$n(\upsilon,t) = E(t-\upsilon)\bar{l}(\upsilon,t), \tag{5}$$

where $\bar{l}(\upsilon,t)=1$ is the labor hours worked and $E(t-\upsilon)$ is the so-called efficiency index (Bettendorf and Heijdra, 2006), which falls exponentially with the worker's age as in Blanchard (1985):

$$E(t-\upsilon) = \omega_0 \mathrm{e}^{-\alpha(t-\upsilon)}, \tag{6}$$

where ω_0, is normalized to 1 and $\alpha > 0$ specifies the speed at which productivity falls with age.

At each moment t the household chooses the paths of consumption and financial assets to maximize lifetime utility (1) subject to budget constraint (2) and a transversality condition. The initial value of the financial assets $a(\upsilon,t)$ and the government consumption per household are taken as given.

The optimal path of household consumption is defined by the Euler equation:

$$\frac{\dot{\bar{c}}^R(\upsilon,t)}{\bar{c}^R(\upsilon,t)} = r(t) - \rho, \tag{7}$$

from which it follows that in each moment consumption is proportional to total wealth:

$$\bar{c}^R(\upsilon,t) = (\rho+\beta)(\bar{a}(\upsilon,t) + \bar{a}_H^R(\upsilon,t)). \tag{8}$$

Human wealth, $\bar{a}_H^R$, is defined as the present value of after-tax labor income:

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[21] See Yaari (1965), Blanchard (1985). Yaari (1965) has introduced a notion of actuarial note, defined as a note that the consumer can buy or sell and which is valid until the consumer dies, at which time it is automatically cancelled.

[22] A dot above the variable signifies the variable's time derivative: $\dot{\bar{a}}(\upsilon,t) = d\bar{a}(\upsilon,t)/dt$.

[23] Heijdra and Romp (2009) analyze the consequences of demographic shocks and changes in pension system on main macroeconomic variables in the OLG model with endogenous labor supply and endogenous retirement age.

$$\bar{a}_H^R(\upsilon,t)=\int_t^{\infty} WI^R(\upsilon,\tau)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau, \tag{9}$$

where $R(t,\tau)=\int_t^{\tau} r(s)ds.$

Non-Ricardian households whose share in the total population is λ, consume all their disposable income. Their consumption, therefore, is defined by (10):

$$\bar{c}^{NR}(\upsilon,t)=WI^{NR}(\upsilon,t)\,. \tag{10}$$

They have the same utility function as Ricardian consumers, defined by equation (1). Their net labor income is identical to that of Ricardian agents.

$$WI^{NR}(\upsilon,t)=\begin{cases}(1-t_L)W^N(\upsilon,t)-\bar{t}_W & \text{for } t-\upsilon\le\pi,\\ (1-t_L)W^N(\upsilon,t)+\bar{z} & \text{for } t-\upsilon>\pi.\end{cases} \tag{11}$$

This type of household consumes all its income, and is unable to save and smooth consumption over its lifetime. Similar to the human wealth of Ricardian agents, we can define the human wealth of non-Ricardian agents. In this case it is equal to the present value of their lifetime consumption.

$$\bar{a}_H^{NR}(\upsilon,t)=\int_t^{\infty} WI^{NR}(\upsilon,\tau)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau, \tag{12}$$

2.2.Demography and aggregate household sector

The framework allows us to consider non-zero population growth, by distinguishing the probability of death $\beta\ge 0$, and the probability of birth, $\eta>0$.[24] The population size $L(t)$, which consists of Ricardian agents $(1-\lambda)$ and non-Ricardian agents (λ), and grows proportionally with the net growth rate n_L:

$$\frac{\dot{L}(t)}{L(t)}=\eta-\beta=n_L. \tag{13}$$

Taking into account a normalization $L(0)=1$, the population size is:

$$L(t)=\mathrm{e}^{n_L t}. \tag{14}$$

The size of the generation born at t is proportional to the current size of the population:

$$L(\upsilon,\upsilon)=\eta L(\upsilon). \tag{15}$$

The size of each generation falls exponentially with the probability of death β:

$$L(\upsilon,t)=\ \mathrm{e}^{\beta(\upsilon-t)}L(\upsilon,\upsilon),t\ge\upsilon. \tag{16}$$

The current size of the generation born at time υ can be obtained by substituting (14) and (15) into (16):

$$L(\upsilon,t)=\eta\mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}. \tag{17}$$

[24] This framework was developed by Buiter (1988).

The aggregate variables are defined as the integral of the variable values, specific for each living generation, weighted by the size of that generation. Aggregate consumption can be defined as follows:

$$C(t)=\int_{-\infty}^{t} L(\upsilon,t)\bar{c}(\upsilon,t)d\upsilon=\lambda C^{K}(t)+(1-\lambda)C^{R}(t), \tag{18}$$

where $L(\upsilon,t)$ and $\bar{c}(\upsilon,t)$ are given by (8), (10) and(17), respectively.

Aggregating (8) by generations of Ricardian consumers we obtain (19) which postulates that aggregate consumption of Ricardian agents is proportional to their wealth, where $A(t)$ is aggregate financial wealth and $A^{H}(t)$ is aggregate human wealth:

$$C^{R}(t)=(\rho+\beta)\left[A(t)+A_{H}^{R}(t)\right]. \tag{19}$$

The growth rate of aggregate consumption is obtained from (18), taking into account (7) and (17):

$$\frac{\dot{C}^{R}(t)}{C^{R}(t)}=\left[r(t)-\rho\right]+\frac{\eta L(t)\bar{c}^{R}(t,t)-\beta C^{R}(t)}{C^{R}(t)}, \tag{20}$$

where $r(t)-\rho$ is the growth of individual consumption of Ricardian agents, while the second term represents the so-called generational turnover (Bettendorf and Heijdra, 2006), which depends on the demographic parameters. Aggregate consumption of Ricardian agents increases with the arrival of new agents and decreases with the death of the older generation. The growth rate of the aggregate consumption of Ricardian agents can be simplified to:[25]

$$\frac{\dot{C}^{R}(t)}{C^{R}(t)}=r(t)-\rho+\alpha+n^{L}-(\rho+\beta)\frac{\eta\gamma L(t)+(\alpha+\eta)A(t)}{C^{R}(t)}, \tag{21}$$

$$\gamma(t)=\frac{-d(t)}{r(t)+\beta}+\left(r(t)+\alpha+\beta\right)\left(\frac{\mathrm{e}^{-\beta\pi}}{1-\mathrm{e}^{-\eta\pi}}\right)\left(\frac{z^{R}-d(t)}{r(t)+\beta}\right)\left(\frac{\mathrm{e}^{-r(t)\pi}-\mathrm{e}^{-n_{L}\pi}}{n^{L}-r(t)}\right). \tag{22}$$

The aggregate consumption growth of Ricardian agents exceeds the growth of their individual consumption ($r(t)-\rho$) if net population growth is positive ($n_{L}>0$), and labor productivity decreases over time ($\alpha>0$). At the same time, aggregate consumption growth of Ricardian agents can be lower than the individual consumption growth if newborns consume less or if the redistribution of wealth from the young to the old through the pension system takes place. In contrast to Bettendorf and Heijdra (2006), γ depends on the deficit of the pension fund ($d(t)$ – per capita deficit of the pension fund) which is defined below.[26]

Bettendorf and Heijdra (2006) point out that $\eta\gamma/(r+\alpha+\beta)$ in an economy with one type of consumer can be considered as the per capita deficit of the pension system. When $r>n_{L}$ social contributions are perceived by working households as a tax on labor income, since they are forced to save at the rate n_{L}, which is lower than the market rate r. For retirees the opposite effect takes place.

[25] For more details see Appendix 2.
[26] Deficit of the pension fund is defined by the difference between the sum of pensions paid by the Pension Fund and the sum of social contributions paid to the Pension Fund (see equation 39).

Aggregate financial wealth owned by Ricardian consumers is defined as follows:

$$A(t) = \int_{-\infty}^{t} L(\upsilon,t)\bar{a}(\upsilon,t)d\upsilon. \tag{23}$$

The definition of aggregate savings can be found by differentiating equation (23) for the aggregate financial wealth with respect to time and taking into account that the newborn generation does not have any financial wealth, $\bar{a}(t,t) = 0$:

$$\dot{A}(t) = -\beta A(t) + \int_{-\infty}^{t} L(\upsilon,t)\dot{\bar{a}}(\upsilon,t)d\upsilon. \tag{24}$$

By substituting (2) to (24) we get:[27]

$$\dot{A}(t) = r(t)A(t) + WI(t) - C^{R}(t), \tag{25}$$

$$WI(t) = \frac{\eta\omega_0}{\alpha+\eta}(1-t_L)F_N(k_N(t),1))L(t) + D(t), \tag{26}$$

where $F_N(k_N(t),1)$ is the marginal product of labor and $D(t)$ is the deficit of the pension system, based on the contributions and pensions of Ricardian agents.

The aggregate labor supply at time t measured in efficiency units is proportional to the population size in the corresponding period. It is obtained from (5), (6), (14) and (17):

$$N(t) = \int_{-\infty}^{t} L(\upsilon,t)\bar{n}(\upsilon,t)d\upsilon = \frac{\eta\omega_0}{\alpha+\eta}L(t). \tag{27}$$

2.3. Firms

As opposed to Bettendorf and Heijdra (2006), we consider a closed economy with an endogenous interest rate, which is important in the estimation of pensions. The output is produced according to the Cobb-Douglas technology:

$$Y = F(K,N) = K^{\varepsilon}N^{1-\varepsilon}, \tag{28}$$

where K and N represent capital and effective units of labor.

Producers maximize profit, choosing the optimal level of capital and labor:

$$\Pi(t) = Y(t) - \int_{-\infty}^{t} W^{N}(\upsilon,t)L(\upsilon,t)d\upsilon - W^{K}(t)K(t), \tag{29}$$

where $W^{K}(t)$ is a capital rent and $W^{N}(\upsilon,t)$ is the wage at time t of the worker of generation υ.

The first order conditions are:

$$W^{K}(t) = F_K(k_N(t),1), \tag{30}$$

$$W^{N}(t) \equiv \frac{W^{N}(\upsilon,t)}{E(\tau-\upsilon)} = F_N(k_N(t),1), \tag{31}$$

[27] For more details see Appendix 2.

where $F_K = \partial F / \partial K_N$ and $F_N = \partial F / \partial N$. $W^N(t)$ is the wage per unit of effective labor and $k_N(t) = K(t)/N(t)$ is the capital effective-labor ratio. The produced output is allocated to the total private consumption C, investment I and government expenditures G.

$$Y(t) = C(t) + I(t) + G(t). \tag{32}$$

The optimal investment decision is based on the maximization of the net present value of cash flows from the investor's capital stock subject to the capital accumulation identity:

$$V(t) = \int_t^{\infty} \left[W^K(\tau) K(\tau) - I(\tau) \right] \mathrm{e}^{-R(t,\tau)} d\tau, \tag{33}$$

$$\dot{K}(t) = I(t) - \delta K(t), \tag{34}$$

where $R(t,\tau) = \int_t^{\tau} r(s) ds$ is the discount factor.

The first order condition (35), specifies that the rental rate W^K equals the return on capital $r(t)$ taking into account the capital depreciation rate δ.

$$W^K(t) = r(t) + \delta. \tag{35}$$

2.4. Public sector

Government budget identity defines the accumulation path of public debt $A^G(t)$, which depends on the current government expenditures $G(t)$, labor tax revenues and additional expenditures, which we assume come from covering the total deficit of pension fund $D(t)$. It can be written as follows:

$$\dot{A}^G(t) = r(t) A^G(t) + G(t) - t_L W^N(t) N(t) + D(t). \tag{36}$$

Taking into account the transversality condition:

$$\lim_{\tau \to \infty} A^G(\tau) \mathrm{e}^{-R(t,\tau)} = 0, \tag{37}$$

the public debt is:

$$A^G(t) = \int_t^{\infty} \left[t_L W^N(\tau) N(\tau) - G(\tau) - D(\tau) \right] \mathrm{e}^{-R(t,\tau)} d\tau. \tag{38}$$

The key difference with the paper of Heijdra and Bettendorf (2006) is the assumption that the PAYG pension system can be run on an unbalanced-budget basis, with a deficit $D(t)$:

$$t_W (1 - e^{-\eta\pi}) L(t) = z\mathrm{e}^{-\eta\pi} L(t) - D(t). \tag{39}$$

The left-hand side of (39) represents the total social contributions paid by the young generation, while on the right-hand side are total pensions paid to the old and the deficit (or surplus) of the pension fund if the sum of social contributions and pensions do not match. For easier comparison, we assume that the government determines the value of social contributions by setting the value of ψ, which is the share of the median wage that goes to social contributions, paid until retirement. This makes the value of social contributions depend on the retirement age:

$$t_W = \psi \omega_0 F_N(k(t),1)\ \mathrm{e}^{-\alpha\left(\pi - \frac{1}{\alpha}\ln\left(\frac{1+\mathrm{e}^{\alpha\pi}}{2}\right)\right)}. \tag{40}$$

The value of pensions can be expressed the same way, as a share of the median wage that goes to pensions, ζ :

$$z = \zeta \omega_0 F_N(k(t),1)\ \mathrm{e}^{-\alpha\left(\pi - \frac{1}{\alpha}\ln\left(\frac{1+\mathrm{e}^{\alpha\pi}}{2}\right)\right)}. \tag{41}$$

2.5. Model summary and social welfare

Table 1 below summarizes the key equations of the model in per capita terms, where the endogenous variables are k, y, g, c^T, c^R, c^{NR}, a, a^G, r, W^N, W^K, γ, n, t_W, z. The parameters are β, η, n_L, α, δ, κ,λ and ρ. The policy instruments are the retirement age π, the share of the median wage that goes to social contributions and pensions, ψ and ζ, public expenditures as a share of output, θ, and income tax t_L.

Table 1. Summary of the Model

Description	Analytical representation	
Dynamic equations:		
Capital	$\dot{k}(t) = ny(t) - c^T(t) - g(t) - (n_L + \delta)k(t)$	(T1.1)
Private consumption	$\dot{c}^R(t) = (r(t) - \rho + \alpha)c^R(t) - (\rho + \beta)(\eta\gamma(t) + (\alpha + \eta)a(t))$	(T1.2)
Public debt	$\dot{a}^G(t) = (r(t) - n_L)a^G(t) + g(t) - nt_L W^N(t) + d(t)$	(T1.3)
Assets	$\dot{a}(t) = (r(t) - n_L)a(t) - c^R(t) - n(1-\lambda)(1-t_L)W^N(t) + d(t)$	(T1.4)
Static equations:		
Pension fund	$\gamma(t) = \frac{-d(t)}{r(t)+\beta} + \left(r(t)+\alpha+\beta\right)\left(\frac{e^{-\beta\pi}}{1-e^{-\eta\pi}}\right)\left(\frac{z-d(t)}{r(t)+\beta}\right)\left(\frac{e^{-r(t)\pi} - e^{-n_L\pi}}{n^L - r(t)}\right)$, where $t_W(1-e^{-\eta\pi}) = ze^{-\eta\pi} - d(t)$	(T1.5)
Rental rate	$W^K(t) = \varepsilon k_N(t)^{\varepsilon-1} = \varepsilon\left(\frac{k(t)}{n}\right)^{\varepsilon-1}$	(T1.6)
Interest rate	$r(t) = W^K(t) - \delta$	(T1.7)
Wage	$W^N(t) = (1-\varepsilon)y(t)$	(T1.8)
Output	$y(t) = k_N^{\varepsilon}(t) = \left(\frac{k(t)}{n}\right)^{\varepsilon}$	(T1.9)
Supplied efficiency units	$n = \frac{\eta\omega_0}{\alpha+\eta}$	(T1.10)
Wealth	$a^R(t) = k(t) + a^G(t)$	(T1.11)
Total consumption	$c^T(t) = c^R(t) + c^{NR}(t)$	(T1.12)

Equation T1.1 corresponds to the accumulation of per capita capital, and it is obtained by combining (34) and (35). Equation T1.2 represents an optimal path of consumption per capita, obtained from (21) in per capita terms. Equation T1.3 is the government budget constraint expressed in per capita terms, derived from the government budget constraint (36). The last dynamic equation, Equation T4, represents the accumulation of per capita assets and is obtained from (25), taking into account (26) and (39).

Definition 1. Given the set of policy variables $\{\pi, \psi, \zeta, \theta, t_L\}$ that satisfy the government budget constraint, the set $\{W_t^K, r_t, y_t, c_t^T, c_t^R, c_t^{NR}, k_t, a^R{}_t, a_t^G, d_t\}$ defines equilibrium, if it satisfies the optimal conditions of households and firms, (7) and (30)-(31), and equilibrium conditions for goods and capital markets. However, according to Walras' Law only two market equilibrium conditions are needed to ensure equilibrium on these markets:

$$y(t) = c^T(t) + i(t) + g(t), \tag{41}$$

$$a(t) = k(t) + a^G(t). \tag{42}$$

The system of dynamic equations is non-linear and cannot be solved analytically, so we are solving the system of equations T1.1-T1.3 numerically, taking into account T1.12.[28]

To distinguish the socially optimal policy mix of measures (income tax and social contributions) we check whether the resulting set of possible equilibria satisfies the stability condition of the equilibrium.[29]

Extending the model further to incorporate welfare analysis, we define the social welfare function as the present value of the utility of all currently living and future generations weighted by their share of the population. Social welfare is a sum of the welfare of young generations and retirees.

$$\begin{aligned} SW(t) &= \lambda \int_t^{\infty} \int_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[(1-\kappa)\ln \bar{c}^{NR}(\upsilon,\tau) + \kappa \ln g] e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau + \\ &+ (1-\lambda) \int_t^{\infty} \int_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[(1-\kappa)\ln \bar{c}^{R}(\upsilon,\tau) + \kappa \ln g] e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau + \\ &+ \lambda \int_t^{\infty} \int_{\tau-\pi}^{\tau} L(\upsilon,\tau)[(1-\kappa)\ln \bar{c}^{NR}(\upsilon,\tau) + \kappa \ln g] e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau + \\ &+ (1-\lambda) \int_t^{\infty} \int_{\tau-\pi}^{\tau} L(\upsilon,\tau)[(1-\kappa)\ln \bar{c}^{R}(\upsilon,\tau) + \kappa \ln g] e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau. \end{aligned} \tag{43}$$

Taking into account equation (8) and (10), individual consumption of the young and old generations are expressed as functions of their individual human wealth, which depends on the effective capital.[30] Simplifying equation (43) the following welfare function can be

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[28] All calculations were conducted using Matlab. Setting $\dot{k}(t) = 0$, and $\dot{a}^G(t) = 0$, we use a root-finding method (the bisection method) to define the level of capital per capita to bring the growth of per capita consumption to zero, $\dot{c}^R(t) = 0$. All possible combinations of feasible model parameters and policy instruments are considered to determine the steady-state level of k^*.

[29] The stability condition ensures that the determinant of the Jacobian matrix of the log-linearized dynamic system of equations T1.1, T1.2 and T1.4 is negative. In this case the model is locally saddle-point stable. The constraint on the public debt ensures that public debt does not exceed 1.5 times the output.

[30] For more details see Appendix 3.

obtained, where a_H^o and a_H^y is the human wealth of the young and old generations of both types of consumers:

$$SW(t) = \chi\, \mathrm{e}^{-\eta\pi} \left[(\ln((\rho+\beta)a_H^o) + \zeta(\pi\eta+1)/\eta) + \kappa \ln g \right] - \\ -\chi\left[(1-\mathrm{e}^{-\eta\pi})(\ln((\rho+\beta)a_H^y) - \kappa \ln g) - \eta^{-1}\zeta(1-\kappa)(1-\mathrm{e}^{\eta\pi} - \eta\pi\mathrm{e}^{-\eta\pi})) \right], \tag{44}$$

$$\chi = \frac{\mathrm{e}^{n_L t}}{n_L - \rho - \beta}, \zeta = r - \rho.$$

It is known that in the OLG model with Ricardian agents and a dynamically efficient equilibrium, a PAYG pension system worsens social welfare.[31] If the level of pensions is chosen optimally from the maximization of social welfare, it would be optimal to set zero pensions and social contributions. Taking this result into account in the maximization problem the level of pensions and the retirement age were fixed in order to analyze the optimal choice of income taxes and social contributions.

It is worth mentioning that the equilibrium with diminishing labor productivity can be dynamically inefficient if the speed of the decrease in productivity, α, is large enough. In this case labor income is high during the youth and falls rapidly with age, so agents save a lot during youth which makes capital stock too large, leading to an overaccumulation of capital. The necessary condition for dynamic efficiency is $\alpha < \rho$, which corresponds to a positive interest rate. The calibration used here satisfies the condition of dynamic efficiency.

2.6. Calibration

The optimal choice of the retirement age, social contributions and pensions illustrates that the pension system is redundant. This result coincides with the results of neoclassical models, with forward-looking agents having an incentive to save on their own in order to smooth their consumption. That is why we investigate the optimal mix of income tax and social contributions with a fixed level of pensions and a fixed retirement age. Table 2 contains the parameters used for the baseline calibration.

[31] See Blanchard and Fischer (1989).

Table 2. Baseline calibration

Variable	Symbol	Value	Source
Rate of time preference	ρ	0.015	Heijdra, Ligthart (2006)[32]
Birth rate	η	0.0125	Heijdra, Ligthart (2006)[33]
Probability of death	β	0.0125	Heijdra, Ligthart (2006)
Output elasticity of capital	ε	0.33	Standard[34]
Speed of decline in the labor efficiency	α	0.0125	Nickel et al. (2008)[35]
Capital depreciation rate	δ	0.03	Heijdra, Ligthart (2006) [36]
Retirement age	π	60	Bettendorf, Heijdra (2006)[37]
Share of non-Ricardian agents	λ	[0; 0.9]	A range of possible values

The share of pensions, ζ, is fixed at 30% of the median lifetime wage, while the optimal size of the mandatory social contributions, ψ, as a share of the median wage, and income tax are chosen optimally from the maximization of social welfare.[38] The share of government expenditures, θ, is fixed at 25% of GDP, a common value for the OECD countries. The productivity declines with age at the speed of α, which equals 1.25%, meaning that the worker is half as productive at retirement age than he was at the beginning of life. For the results presented below, the death rate β equals 1.25%, giving a life expectancy of 80 years. The birth rate η varies from 1% to 3%. This range allows us to analyze both negative and positive population growth. In considered steady states the endogenous interest rate is higher than the population growth (the so-called Aaron condition is satisfied).

There is no consensus in the literature concerning the number of non-Ricardian agents in the economy. The estimates vary from 50% in Campbell and Mankiw (1989) for the US to 0.24-0.37 for the Euro area estimated by Coenen and Straub (2005).[39] At the same time Marto (2014) estimated the share of non-Ricardian agents as equal to 0.58 for the Portuguese economy. As for Russia, Tiffin and Hauner (2008) assume that 40% of Russian households are liquidity-constrained in the GIMF model calibrated for Russia.[40]

In the next two sections we consider the optimal fiscal instruments in an economy where all agents are Ricardian (Section 3) and in an economy with two types of agents, Ricardian and non-Ricardian (Section 4).

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[32] The value belongs to the interval considered by Heijdra and Ligthart (2006): from 0.01 to 0.04.
[33] In Heijdra and Ligthart (2006) the birth rate varies from 0.01 to 0.04, while the death rate varies from 0 to 0.03. In the baseline calibration the birth rate equals 0.015 and the death rate is 0.01
[34] In Heijdra and Ligthart (2006) elasticity with respect to capital equals 0.35.
[35] It is consistent with the value used in Nickel et al. (2008) where this parameter equals 0.014, death rate at 0.01.
[36] This value is consistent with the literature. Heijdra and Ligthart (2006) use depreciation rate which is equal to 0.06 with the higher rate of time preference being equal to 0.04. In Heijdra et al. (2017) capital depreciation rate equals to 0.036.
[37] Although Bettendorf and Heijdra (2006) do not use calibration, in the numeric example the restriction on the retirement age postulates that it should exceed 48.5 or 56.8 depending on the birth and death rates considered.
[38] The results are robust to the change in the share of the median wage paid to pensions.
[39] Here the estimates for posterior means are provided. These estimates are rather low, taking into account the fact that Coenen and Straub (2005) used the value of 0.5 as a mean of the prior distribution.
[40] Global Integrated Monetary and Fiscal model (GIMF), presented in Kumhof, Laxton (2007, 2009, 2013), Kumhof et al. (2009, 2010).

3. Optimal fiscal policy instruments in a homogeneous economy

This section first shows how optimal social contributions and income tax rate depend on the retirement age and population growth. We start by searching for the optimal proportion of the median wage that goes to social contributions, and move to the determination of the choice for both social contributions and income tax rate. In Subsection 3.2 the optimal choice of these two instruments is considered under a balanced and unbalanced pension system. Subsection 3.3 provides a robustness check, covering the optimal choice of fiscal instruments under different life expectancy and labor productivity.

3.1. Policy instruments under unbalanced pension system depending on the retirement age

In order to investigate how the optimal social contributions depend on the retirement age we first consider the case with a fixed income tax ($t_L = 40\%$). Quantitative results suggest that a higher retirement age leads to a lower rate of social contributions ψ (Figure 1). The value of social contributions t_W is also affected by the change in the median wage. Median wage increases with the higher retirement age due to the higher per capita capital and decreases with the higher retirement age due to the decreasing productivity of labor. The latter effect is stronger, so median wage decreases with the higher retirement age. As a result, lower pensions correspond to a higher retirement age. The optimal level of ψ decreases with higher population growth as it leads to a higher share of young generation members who pay social contributions.

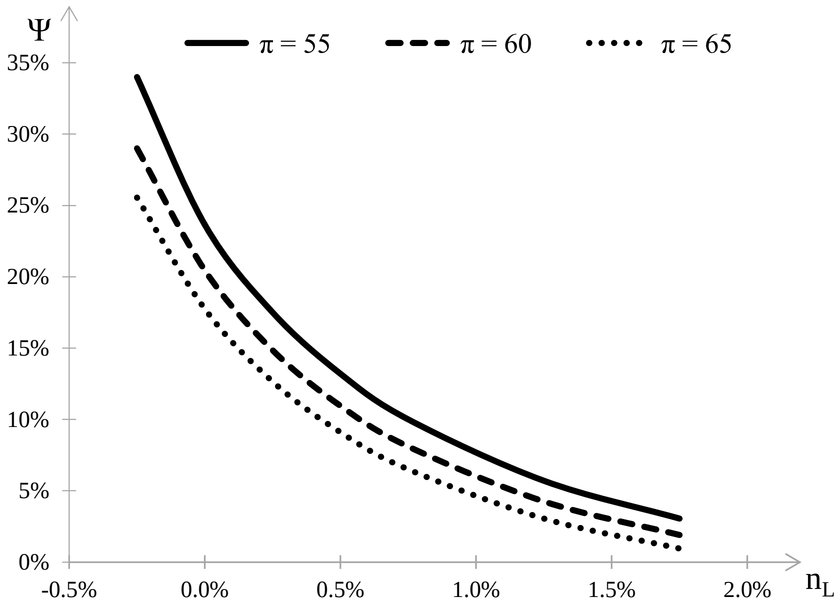

Figure 1. Optimal social contributions rate under different retirement age

Source: estimates based on the model.

Next, we consider the optimal choice of two instruments: income tax, t_L, and the rate of social contributions, t_W. In this case the corner solution takes place: the optimal income tax rate is positive, while the rate of social contributions equals zero. The corner solution takes place because zero social contributions leads to a lower optimal welfare improving income tax rate. Other steady states with zero social contributions and a higher income tax rate are characterized by lower social welfare. In the neighborhood of optimal instrument combination the same level of capital per effective unit of labor can be reached with zero contributions and positive income tax as well as with small positive contributions and lower income tax. However, this equilibrium is characterized by higher public debt and a smaller deficit of the pension fund. Household savings are invested in the public debt and not in capital, which leads to inefficiency. As a result, the optimal combination consists of zero social contributions and the minimum equilibrium level for the income tax rate.

The results of welfare maximization for $\pi = 60$ and $\pi = 65$ are presented in Table 3 below.[41]

Table 3. Optimal income tax under different population growth and retirement age

	n_L	**-0.25%**	**0%**	**0.25%**	**0.5%**	**0.75%**	**1.25%**	**1.75%**
				$\pi = 60$				
Income tax rate	t_L	50.3%	48%	46.5%	45.3%	44.2%	42.4%	41.1%
Pension fund deficit to output	d_y	8.1%	7%	6%	5.2%	4.4%	3.3%	2.4%
				$\pi = 65$				
Income tax rate	t_L	48.9%	47.2%	45.7%	44.5%	43.4%	41.8%	40.6%
Pension fund deficit to output	d_y	7.5%	6.4%	5.4%	4.6%	3.9%	2.8%	2.1%

The pension system in this case is run with a deficit (d_y - the share of the deficit in output). It decreases with population growth and the retirement age. Pensions are set at 30% of the median wage, which changes with the longer working period, so pensions adjust automatically to the changes in the retirement age.

Figure 2 shows how an optimal income tax depends on the population growth. Optimal income tax exhibits the same inverse relationship as social contributions (Figure 1). In this case social contributions are zero, while the income tax rate is decreasing with higher population growth: the deficit and pensions can be financed with lower income taxes when the share of the young population is high.

[41] Different level of birth rates are considered with constant death rate, $\beta = 1.25\%$. This leads to different levels of population growth n_L.

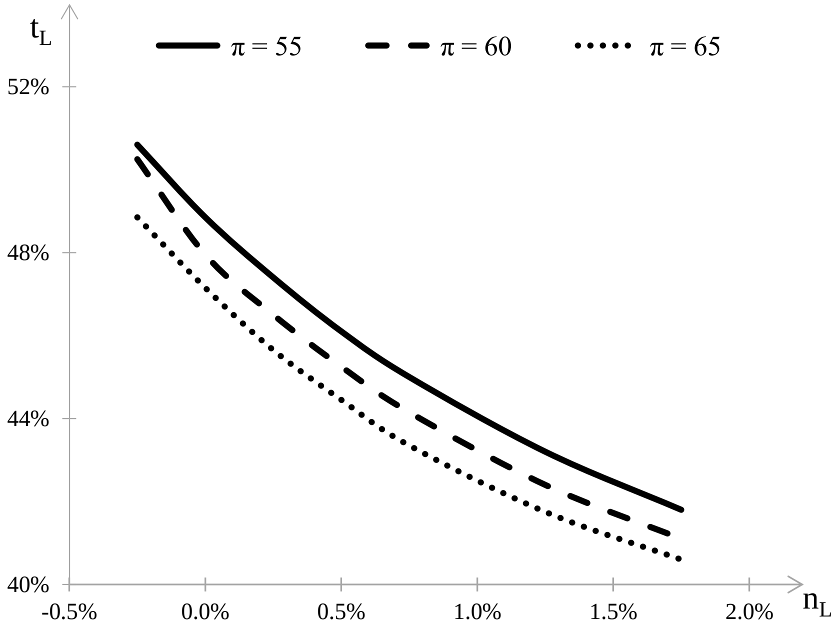

Figure 2. Optimal income tax under different retirement age

Source: estimates based on the model.

Higher income tax and zero social contributions, when they both are chosen optimally, illustrate that these instruments can be considered as substitutes in financing pensions. Additional calculations show that an increase in ψ by 0.5% leads to the decrease in the optimal income tax rate by 0.2%. However, combinations of lower income tax and positive social contributions result in lower level of capital per capita. Per capita capital is more sensitive to the change in the rate of social contributions than to the change in the income tax rate. The higher sensitivity of capital per capita to the income tax rate is explained, first, by the shorter payment period of social contributions (while income tax is paid throughout the life, social contributions are paid only up to the retirement age) and, second, by the fact that social contributions are defined by the median wage and paid until the retirement age.

3.2. Optimal policy under balanced and unbalanced pension system

Next, we consider how the optimal policy mix changes depending on whether the pension system is balanced or not. First, under the balanced pension system the optimal income tax rate is lower than in the case of the unbalanced pension system for all growth rates (Figure 3), because in this case pensions are financed by social contributions and not out of the government budget. Second, under a balanced pension system the optimal level of income tax is constant, while the share of social contributions decreases with population growth. A higher birth rate increases the value of the social contributions to the pension fund because in this case more members of the young generation make contributions. Thus, the lower social contribution rate is needed to keep the pension fund balanced.

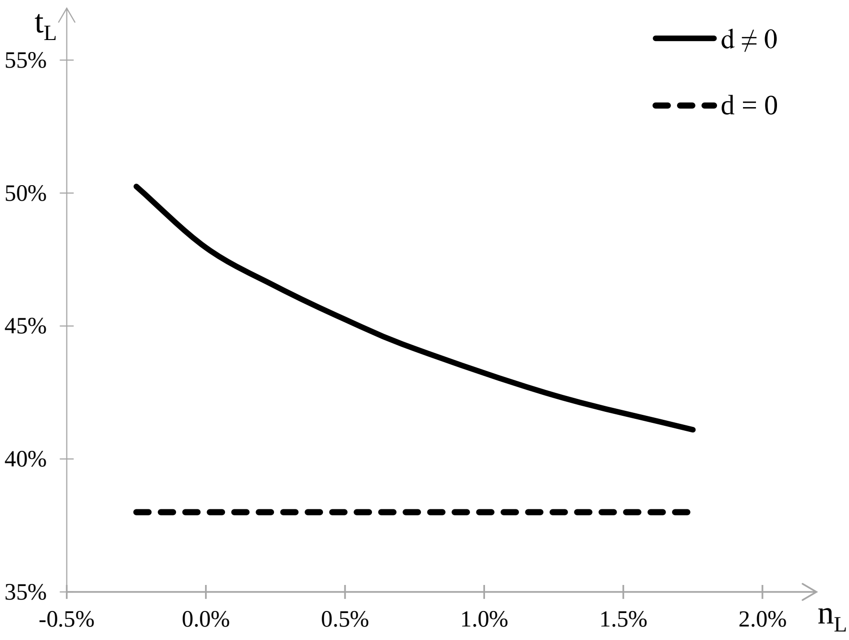

Figure 3. Optimal income tax under balanced and unbalanced pension system

Source: estimates based on the model.

Therefore, depending on whether the pension system is balanced or not, either the optimal income tax or social contributions change with the growth rate, while the other instrument remains at its optimal level. Figure 4 illustrates the results for the optimal level of ψ for a balanced and an unbalanced pension system.

In the case of a balanced pension system, social welfare is lower than in the case of an unbalanced pension system. A balanced pension system is accompanied by a higher level of per capita capital and therefore, higher social welfare. This finding corresponds to the results of the OLG models with a dynamically efficient steady state where a PAYG pension system leads to lower social welfare.

An equilibrium with an unbalanced pension system leads to higher social welfare and to a lower level of public debt due to the lower surpluses. This result is explained by the comparison of budget surpluses, which serve as a provision of public debt. Under a balanced pension system tax revenues are lower. First, the optimal tax rate is lower as pensions are financed by social contributions. Second, the aggregate wage is lower due to the lower capital per capita. Public spending is also lower in the case of a balanced system: due to both lower government expenditures and zero deficit of the pension fund.

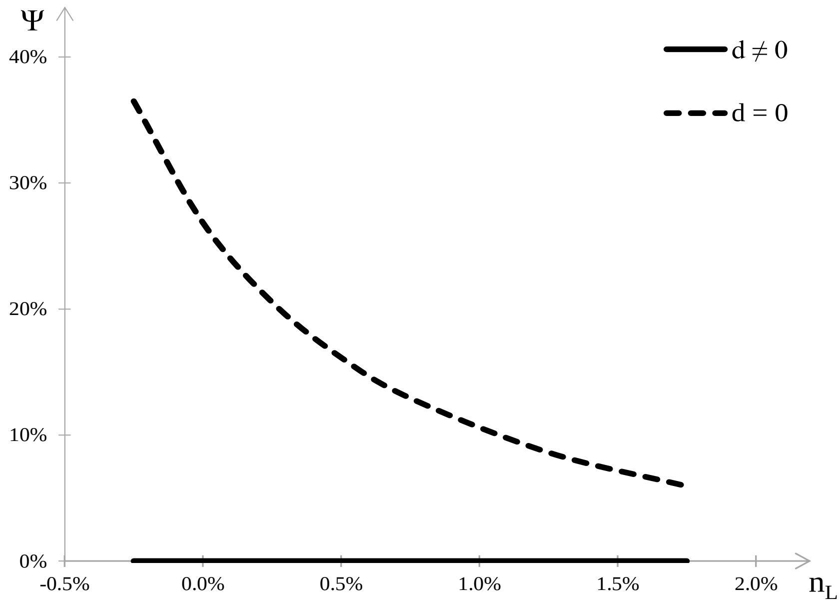

Figure 4. Optimal social contributions under balanced and unbalanced pension system

Source: estimates based on the model.

3.3. Robustness check: life expectancy and labor productivity

The robustness check includes an analysis of how the optimal policy mix of social contributions and income tax is affected by the changes in life expectancy and labor productivity. We compare steady states under life expectancy which is equal to 70 and 80 years under the unbalanced pension system. These levels of life expectancy correspond to β being equal to $\beta = 1.43\%$ and $\beta = 1.25\%$. Income tax and social contribution rates are chosen optimally to maximize the social welfare.[42] We have considered a fixed retirement age so that equilibria under the same birth rates were comparable. The optimal policy mix remains the same: positive rate of income tax and zero social contributions (Table 4).

[42] To check the robustness of results the change in β was considered for different retirement ages, for π from 55 to 70. Since the results for different retirement ages are similar, we provide here only the results for $\pi = 60$.

Table 4. Optimal income tax under different birth rates and life expectancy

	η	**1%**	**1.25%**	**1.5%**	**1.75%**	**2%**	**2.5%**	**3%**
				$\beta = 1.25\%$				
Population growth	n_L	-0.25%	0%	0.25%	0.5%	0.75%	1.25%	1.75%
Income tax rate	t_L	50.3%	48%	46.5%	45.3%	44.2%	42.4%	41.1%
Ratio of pension fund deficit to output	d_y	8.1%	7%	6%	5.2%	4.4%	3.3%	2.4%
				$\beta = 1.43\%$				
Population growth	n_L	-0.43%	-0.18%	0.07%	0.32%	0.57%	1.07%	1.57%
Income tax rate	t_L	50.3%	48%	46.5%	45.3%	44.2%	42.4%	41.1%
Ratio of pension fund deficit to output	d_y	8.1%	7%	6%	5.2%	4.4%	3.3%	2.4%

Under an unbalanced pension system, the higher life expectancy leads to higher level of per capita capital, lower share of per capita private consumption in the output due to the higher output and higher government expenditure in the steady state (defined as a fixed share in the output). Public debt is higher under higher life expectancy due to the higher government expenditure and lower interest rate, while the optimal income tax rate is higher in general in the steady states with higher β.[43] Overall, comparison of the steady states under the same birth rate and different death rate it not generally correct due to the different population growth in the steady states.

The pension system is run with a deficit. Its share of GDP is the same under different β because a per capita deficit of the pension fund is defined by the ratio of pensions to GDP is constant (14.7%), while the optimal share of social contributions, ψ, is zero. Although the absolute value of pensions increases with higher life expectancy due to a higher level of capital, its change is proportional to the change in the output (as both output and median wage are functions of capital). That is why a share per capita deficit of the pension fund in the per capita output remains unchanged.

Next, we consider the case of higher labor productivity. It is modelled as the lower speed, α, with which the product of labor decreases with age. Table 5 shows the results for $\alpha = 1.25\%$ and $\alpha = 1.1\%$.

[43] For this case a higher precision was applied for the choice of t_L (the step equals 0.01%).

Table 5. Optimal income tax under different labor productivity

	n_L	**-0.25%**	**0%**	**0.25%**	**0.5%**	**0.75%**	**1.25%**	**1.75%**
					$\alpha = 1.25\%$			
Income tax rate	t_L	50.3%	47.95%	46.5%	45.25%	44.15%	42.4%	41.1%
Ratio of pension fund deficit to output	d_y	8.08%	6.95%	5.99%	5.15%	4.42%	3.27%	2.39%
					$\alpha = 1.1\%$			
Income tax rate	t_L	50.4%	48.3%	46.8%	45.45%	44.35%	42.6%	41.2%
Ratio of pension fund deficit to output	d_y	8.32%	7.17%	6.17%	5.29%	4.55%	3.36%	2.47%

Optimal income tax decreases with population growth and increases with labor productivity. Higher labor productivity results in a higher capital per capita in the steady state, leading to higher output per capita, government expenditures and private consumption. Higher productivity leads to a higher wage, increasing the level of pensions to be equal to 30% of the median wage. It puts higher pressure on the pension system and, thus, the government budget. However, despite the fact that the deficit of the pension fund is higher under higher labor productivity, public debt is lower at equilibrium with the higher productivity due to the higher income tax payments (driven by the higher wage), which have increased more than government expenditures.

4. Optimal fiscal instruments: the role of non-Ricardian agents

In this section we consider the optimal choice of fiscal instruments under different conditions. First, we consider the optimal choice of income tax in an economy without a pension system (zero social contributions and pensions). This case can provide insight into how key variables of the model depend on the share of non-Ricardian agents. We use this steady state as a benchmark to see the consequences of the introduction of a pension system. The pension system is introduced in Subsection 3.2, which provides the results for the optimal choice of income tax and social contributions. In Subsection 3.3 the set of policy instruments is extended to include optimal government expenditures. The set of policy instruments includes income tax, social contributions' share of the median wage and government expenditures' share of total output. However, in Subsections 3.1-3.3 Ricardian and non-Ricardian consumers pay the same social contributions. Subsection 3.4 provides the results for the case of possible discrimination of agents by their type, Ricardian and non-Ricardian. This is modelled by setting different social contributions depending on the type of the agent.

4.1. Optimal income tax

First, let us consider the optimal choice of income tax and the corresponding steady states. In the absence of a pension system, the optimal tax rate, identical for both types of consumers, does not depend on the share of non-Ricardian agents. A tax rate of 15.5% is the lowest interest rate in found equilibria under different parameter combinations (which satisfies the conditions).[44] Equilibria with a higher rate of income tax are characterized by lower investment in capital per capita, which leads to lower welfare. As a result, the lowest feasible equilibrium tax rate is optimal.

The adjustment of the interest rate and public debt ensures the fulfilment of the government budget constraint with the tax rate staying constant. The higher the share of non-Ricardian agents, the lower the level of capital in the economy, as fewer agents can hold savings. The lower level of savings is accompanied by a higher interest rate, which exceeds the population growth. The results are provided in Table 3.6, where a_y^G is the share of public debt in output. The results are calculated with a fixed value of government expenditures at 10% of the output (close to the average optimal level of government expenditures for $\lambda = 0.5$, see Subsection 3.3). As a robustness check we compute the results for $\theta = 0.25$ (see Appendix). The share of government expenditure in the utility $\kappa = 0.3$.[45] Other parameters are equal to the values in baseline calibration ($\eta = 2.5\%$, $\beta = 1.25\%$, $\pi = 60$).

Table 6. Steady states with the optimal choice of income tax rate

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
t_L	15.5%	15.5%	15.5%	15.5%	15.5%	15.5%	15.5%	15.5%	15.5%
k	12.5	12.1	11.7	11.2	10.7	10.1	9.4	8.7	7.9
c_y^T	60.0%	60.6%	61.3%	62.1%	62.9%	64.0%	65.1%	66.4%	68.0%
a_y^G	70.5%	58.6%	48.8%	40.6%	33.8%	28.0%	23.1%	18.9%	15.3%
r	1.7%	1.8%	1.9%	2.1%	2.2%	2.4%	2.7%	3.0%	3.4%

The share of total consumption in output, c_y^T, increases with the higher share of non-Ricardian consumers, as fewer resources are directed to savings. With the value of per capita consumption slowly decreasing with higher λ, the share of consumption increases mainly due to lower per capita output. For commonly used values of the share of non-Ricardian consumers, λ in the range of 40-50%, the model provides an estimate of consumption at about 62%, which conforms to the OECD data.[46]

The allocation of total consumption between the two types of consumers is illustrated by Figure 5. While per capita consumption of non-Ricardian consumers is slowly decreasing with λ, the wage, which defines their consumption, is decreasing with a fall in capital. The interest rate on the savings of Ricardian agents almost doubles with λ rising from 10% to 90%, increasing the income of Ricardian agents. This leads to a significant inequality in consumption of Ricardian and non-Ricardian agents with an increase in the share of non-Ricardian agents.

[44] The choice of tax rate in equilibrium is limited by the equilibrium stability condition (forms lower constraint) and by the restriction that public debt should not exceed 1.5 times GDP, which provides the upper constraint.
[45] When $\theta = 0.25$ the optimal level of income tax is higher due to higher public expenditure.
[46] According to the OECD estimates an average share of final household consumption in GDP for 2010-2016 is 61.02%. For 2016 its estimate is at 60.7%. For the USA the corresponding estimates are 68.36% in 2010-2016 and 68.84% in 2016 (OECD, National Accounts at a Glance, National accounts database).

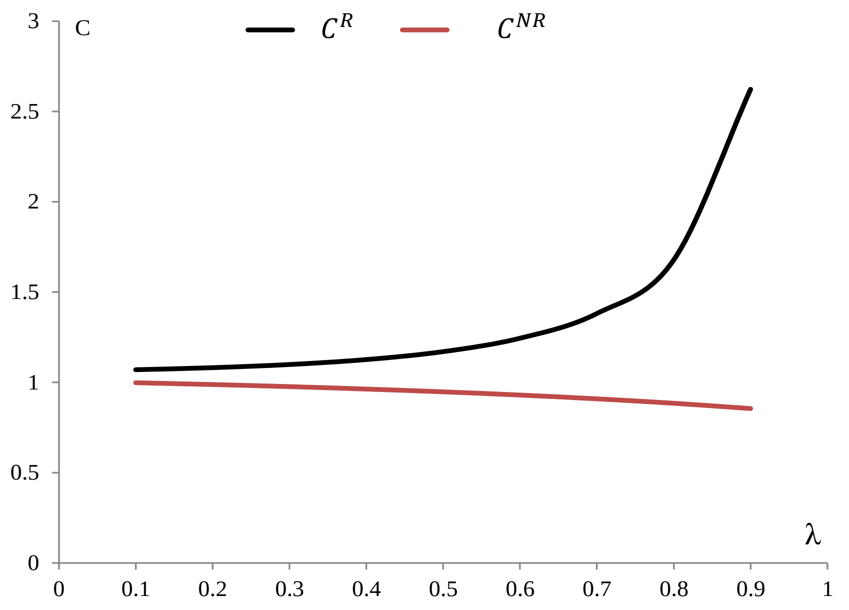

Figure 5. Per capita consumption of Ricardian and non-Ricardian agents as a function of the share of non-Ricardian agents

Source: estimates based on the model.

4.2. Optimal tax instruments under an unbalanced pension system

In subsection 4.1 we considered the choice of the optimal tax rate in the absence of a pension system, now let us consider the choice of income and the share of social contributions which maximize social welfare in an economy with a pension system. The value of pensions for both types of consumers is set at 30% of the median wage. The other parameter values are the same as those used in the previous section. The results are presented in Table 7.[47]

Table 7. Optimal choice of income tax rate and social contributions

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
t_L	20.0%	20.0%	20.0%	20.0%	20.0%	18.5%	0.5%	0.5%	0%
ψ	0%	0%	0%	0%	0%	2.5%	34.0%	34.0%	35.0%

The optimal level of social contributions is zero for a share of non-Ricardian agents λ lower than 50%. This result is similar to the results of Section 3.5, where zero social contributions and positive income tax were obtained in the absence of non-Ricardian agents. However, according to the estimates there exists a threshold level of the share of non-Ricardian consumers (for this calibration it is $\lambda = 55\%$) after which the financing of public spending and pensions is optimal via social contributions instead of via income tax, which

[47] The results for $\theta = 0.25$ are provided in Appendix 2.4. Both the optimal level of income tax and social contributions are higher. A threshold level for λ continues to exist, after which the set of instruments consists of a lower income tax and higher social contributions.

decreases with a higher level of λ.[48] For a low share of non-Ricardian agents income tax revenue and public debt are sufficient to cover government expenditure and pension payments. When the share on non-Ricardian agents is high enough public debt becomes lower as fewer agents (Ricardian type agents) are making savings.

In the economy with a pension system and fixed share of pensions which are financed via income tax and social contributions, the level of social welfare is lower than in the absence of a pension system. Social welfare is lower as a result of a lower level of capital per capita. Higher income tax and positive social contributions imposed after a certain level of λ lead to lower investment in capital. Figure 6 shows that the switch to the positive social contributions after the threshold level of λ leads to a higher difference between levels of capital per capita in the steady state with a pension system and in its absence. In the former case all agents, in addition to the income tax, pay social contributions. As a result, Ricardian agents are able to invest even less in capital.

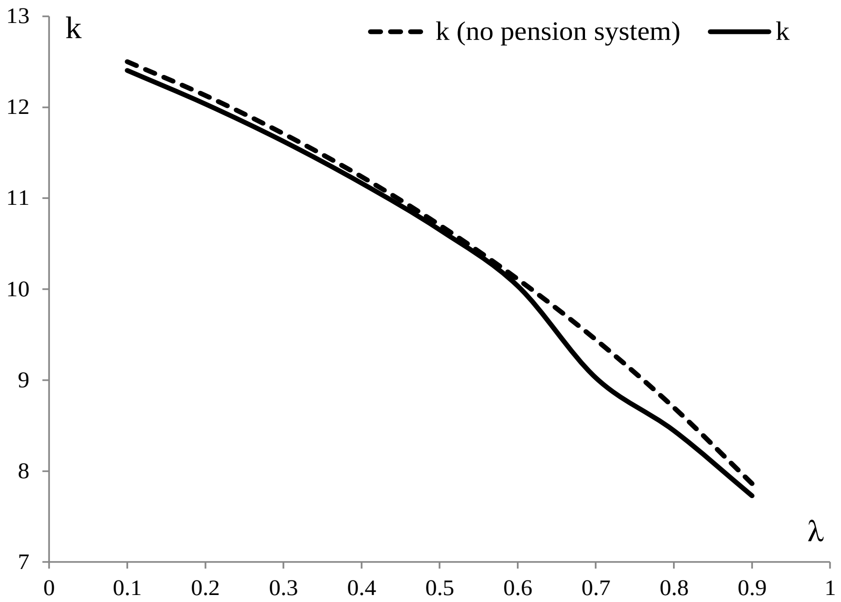

Figure 6. Per capita capital with and without pension system

Source: estimates based on the model.

The lower level of per capita capital (Table 8) results in a slightly lower level of total per capita consumption, c^T (Figure 7).

Table 8. Capital per capita and the share of consumption in output per capita

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
k	12.40	12.04	11.63	11.17	10.65	10.03	8.93	8.38	7.72
k *(no pension system)*	12.50	12.13	11.71	11.24	10.70	10.11	9.44	8.70	7.86
c_y^T	60.2%	60.8%	61.4%	62.2%	63.0%	64.1%	66.0%	67.0%	68.3%

[48] The threshold level for λ is obtained from a maximization of social welfare with a 1% step for λ.

c_y^T *(no pension system)*	60.2%	60.8%	61.4%	62.2%	63.1%	64.1%	66.3%	67.3%	68.4%

The level of social welfare in the economy with optimal fiscal policy and a pension system is lower than in the case without a pension system. In addition to the slightly lower level of per capita consumption, social welfare is affected by a lower level of government expenditure per capita: with the same fixed share of government expenditure of the output (10%), it is lower due to the lower level of per capita output in the economy with a pension system.

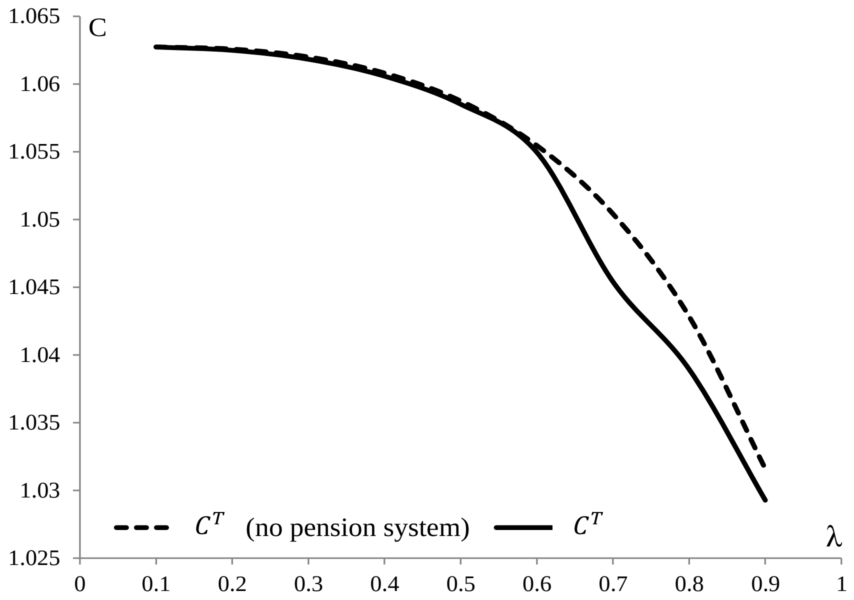

Figure 7. Total per capita consumption with and without pension system

Source: estimates based on the model.

4.3. Optimal public spending and the share of non-Ricardian consumers

In the absence of non-Ricardian agents the optimal choice of size of government expenditure depends on the parameters of the pension system (the value of pensions, social contributions and retirement age) as well as on the income tax rate. However, when the share of government expenditure in the output is fixed it does not adjust to the population structure. On the contrary, in the economy with Ricardian and non-Ricardian agents the government can smooth the utility of non-Ricardian agents, who do not have an opportunity to save for their retirement.

The optimal share of government expenditure grows with the share of non-Ricardian agents (Table 9). When the number of non-Ricardian agents is small (lower than 40%) government expenditures and pensions are financed by the income tax while social contributions for both types of agents are at zero. With an increase in the share of non-

Ricardian agents and, therefore, the share of government expenditure, social contributions for non-Ricardian agents become positive.

The value of government spending is increasing, despite decreasing with λ capital and, thus, output. The impact of the higher share of expenditure exceeds the effect of lower output. The results are presented in Table 9.

Table 9. Optimal public spending, social contributions and income tax

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
θ	1%	1%	3%	6%	9%	11%	15%	18%	21%
t_L	6.5%	6.5%	9.5%	14.0%	18.5%	20.0%	6.0%	10.5%	14.0%
ψ	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%	37.5%	37.5%	39.5%

4.4. Optimal fiscal instruments if discrimination is possible

In this subsection we focus on the case where a different level of social contributions can be imposed on Ricardian and non-Ricardian agents.[49] Let us consider the case where income tax and both values of social contributions are chosen optimally so to maximize the social welfare function. The parameter values are the same as those used in the previous section. The results are presented in Table 10.

Table 10. Optimal choice of income tax rate and social contributions

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
t_L	20.0%	20.0%	19.5%	16.5%	4.5%	1.0%	0%	0%	0%
ψ_{NR}	0%	0%	2.5%	15.0%	54.0%	54.5%	50.0%	43.5%	39.0%

The optimal level of social contributions of Ricardian agents is zero, which is consistent with the results in Section 3.6. For the level of social contributions for non-Ricardian agents the situation is different. According to the estimates there still exists a threshold level for the share of non-Ricardian consumers (for this calibration it is $\lambda = 30\%$) after which the financing of public spending and pensions is optimal via social contributions of non-Ricardian agents instead of via the income tax, which decreases with a higher level of λ.

When different levels of social contributions can be imposed on the agents depending on their type (Ricardian and non-Ricardian), the pension system with a fixed share of pensions and financing via income tax and social contributions leads to a higher level of social welfare. Figure 8 shows that after the threshold level of λ, a higher level of capital per capita is feasible (greater than in case of unified social contributions, Section 3.6.2), as more is invested in capital by Ricardian consumers who do not pay social contributions.

[49] Several adjustments to the baseline model were made to incorporate this assumption. Net income labor is now different among agents due to the different level of social contributions. The deficit of the pension fund is the weighted sum of social contributions by two types of agents and the pensions paid to them.

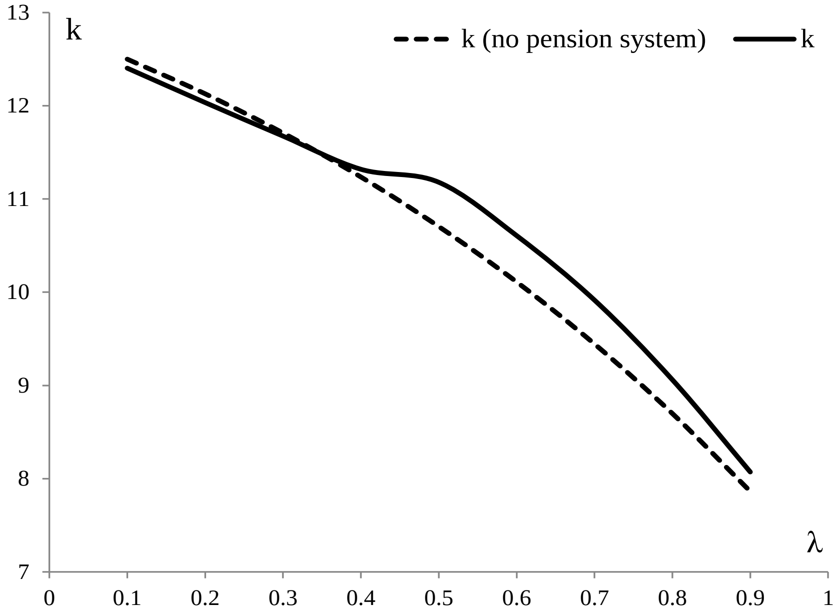

Figure 8. Per capita capital with and without pension system

Source: estimates based on the model.

The higher level of capital per capita results in the higher level of per capita consumption Ricardian agents and the lower consumption of non-Ricardian consumers (Figure 9). The inequality in consumption per capita is higher when agents pay different social contributions. However, the discrimination of agents by their type allows higher social welfare to be achieved for λ higher than the threshold level.

To illustrate the effect of a pension system on households' behavior let us consider individual household consumption profiles. Individual consumption of Ricardian agents is defined according to the equation (8), specifying the level of individual consumption of a Ricardian type household as a share of the household's assets. Individual consumption of non-Ricardian consumers is defined by equation (10) and equals the disposable income of a household of this type. The fact that the labor income of both types of households declines with age (due to the decreasing labor productivity) is taken into account. The value of social contributions and pensions is calculated on the basis of their current labor income.

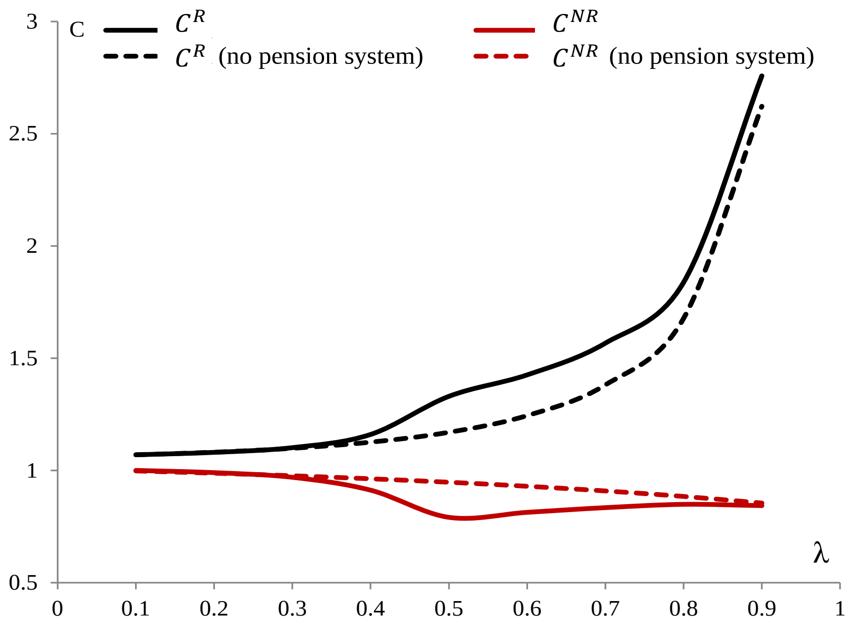

Figure 9. Consumption of Ricardian and non-Ricardian agents with and without pension system

Source: estimates based on the model.

Figure 10 illustrates how net labor income of both types of agents (WI) and the consumption of Ricardian consumers change with the age of the worker. The figure is based on the calibration of the previous section with a fixed level of pensions and the optimal choice of income tax rate, social contributions and public expenditure. WI^{NR} (no pension system) shows the level of disposable income and, therefore, consumption of non-Ricardian agents in the absence of a pension system. In the absence of a pension system disposable income of Ricardian and non-Ricardian agents would coincide: they have the same productivity, which is gradually decreasing with age, and pay the same rate of income tax. Therefore, the disposable income of Ricardian and non-Ricardian agents coincides prior to the retirement age with and without a pension system. In the model with a pension system the level of disposable income of non-Ricardian agents is lower than in the absence of a pension system because in the former case non-Ricardian agents pay social contributions. Figure 10 shows that the introduction of the pension allows the consumption level of non-Ricardian agents after retirement to increase.

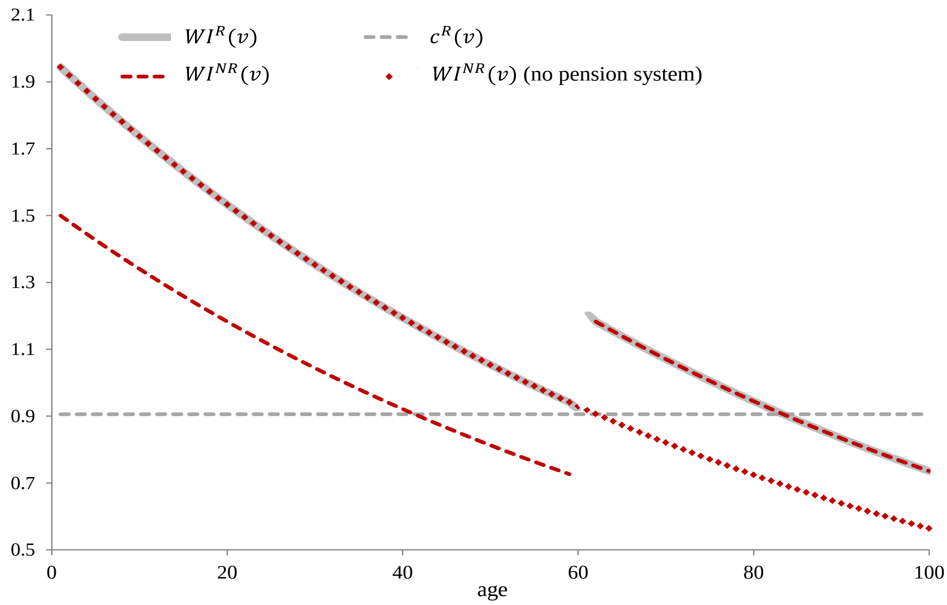

Figure 10. Net labor income and consumption profiles of consumers

Source: estimates based on the model.

5. Conclusion

The OLG model with infinitely living households developed by Bettendorf and Heijdra (2006) was extended by introducing an unbalanced pension system, where the deficit of the pension fund is covered by the transfer from the government budget in the model with an endogenous interest rate. The assumption that liabilities of the pension fund are financed by the government makes the income tax and social contributions act as substitutes in the financing of pensions. During the next stage the model was extended to incorporate two types of consumers: Ricardian consumers, and non-Ricardian consumers, who consume all their disposable income.

First, the model with homogeneous agents was considered. In the case of an unbalanced pension system when both social contributions and income tax are chosen optimally, the optimal policy mix includes a positive income tax rate, which decreases with population growth, and zero social contributions. In the case of a balanced pension system social contributions should be strictly positive to cover the spending of the pension fund. Thus, concerning the combinations of policy instruments, only an internal solution is considered. However, social welfare is lower under a balanced pension system, while the level of public debt is higher, compared to the case of an unbalanced pension system. Under a balanced pension system the optimal income tax is lower than under an unbalanced pension system and does not depend on population growth. Social contributions, on the contrary, decrease with population growth. The results also show that in equilibria with a higher retirement age, optimal social contributions (or income tax under an unbalanced pension system) are lower, as is the public debt. Therefore, pension reforms, namely increase in the retirement age, can be introduced as an additional measure of fiscal consolidation.

Second, on the basis of an extended framework, the way the optimal set of fiscal instruments change with the share of non-Ricardian consumers was shown. When income tax and social contributions are chosen optimally, the optimal rate of social contributions paid by all households becomes positive after a certain threshold level of the share of non-Ricardian consumers, coupled with the lower rate of income tax. Higher taxes lead to a lower level of per capita output. When discrimination among households is possible based on their type, after a certain proportion of non-Ricardian agents in the economy the optimal set of measures includes positive social contributions paid by non-Ricardian agents, decreasing with their income tax rate, and zero contributions paid by Ricardian agents.

An analysis of the optimal choice of social contributions, income tax and public spending shows that the optimal share of public spending increases with the share of non-Ricardian agents to increase their welfare at retirement. The optimal choice of fiscal instruments taking into account the share of non-Ricardian consumers can lead to higher social welfare.

The developed framework can be applied to the analysis of the consequences of demographic changes to public finance and to the development of the optimal consolidation measures. The framework can be extended to analyze a wider set of optimal pension reforms and fiscal policy measures and can be calibrated for a specific country.

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Appendix 1

1. Derivation of the aggregate Euler equation (the case of homogeneous agents)

The equation (18) can be simplified as follows:

$$\frac{\dot{C}(t)}{C(t)} = \left[r(t)-\rho\right] + \frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)}. \qquad \text{(A1.1)}$$

As new generations are born without any financial assets ($\bar{a}(t,t)=0$), thus from (8) $\bar{c}(t,t)=(\rho+\beta)\bar{a}^H(t,t)$ and taking into account (16) we get:

$$\frac{\eta L(t)\bar{c}(t,t)-\beta C(t)}{C(t)} = (\rho+\beta)\frac{\eta L(t)\bar{a}^H(t,t)-\beta(A(t)+A^H(t))}{C(t)}. \qquad \text{(A1.2)}$$

Aggregate human wealth is defined as follows:

$$A^H(t) = \int_{-\infty}^{t-\pi}\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi}d\upsilon + \int_{t-\pi}^{t}\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\le\pi}d\upsilon, \qquad \text{(A1.3)}$$

where:

$$\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi} = \int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau + \int_t^{\infty} z\ \mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau \qquad \text{(A1.4)}$$

and $R(t,\tau)=\int_t^{\tau} r(s)ds.$

$$\begin{aligned}\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\le\pi} &= \int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau - \\ &-\int_t^{\upsilon+\pi} t_W\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau + \int_{\upsilon+\pi}^{\infty} z\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau.\end{aligned} \qquad \text{(A1.5)}$$

For the simplicity of the analysis let us assume the constant interest rate r (so that equations could be applicable to the steady state).

$$\begin{aligned}\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\le\pi} &= \int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{(r+\beta)(t-\tau)}d\tau - \\ &-\frac{t_W}{r+\beta}\left(1-\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right) + \frac{z}{r(t)+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}.\end{aligned} \qquad \text{(A1.6)}$$

We know that age dependent wage can be written as follows:

$$W^N(\upsilon,t) = E(\tau-\upsilon)F_N(k(t),1) = \omega_0\mathrm{e}^{-\alpha(\tau-\upsilon)}F_N(k(t),1), \qquad \text{(A1.7)}$$

$$\int_t^{\infty}(1-t_L)W^N(\upsilon,t)\mathrm{e}^{(r+\beta)(t-\tau)}d\tau=\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t), \tag{A1.8}$$

where $\Omega_0(t)$ is defined as follows:

$$\Omega_0(t)=\omega_0\int_t^{\infty}(1-t_L)F_N(k(t),1)\mathrm{e}^{(r+\alpha+\beta)(t-\tau)}d\tau. \tag{A1.9}$$

Substituting this definition into the expressions of human wealth of workers and retirees, noted above, we get:

$$\begin{aligned}\int_{-\infty}^{t-\pi}L(\upsilon,t)\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi}d\upsilon&=\int_{-\infty}^{t-\pi}\eta\,\mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}\left[\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t)\upsilon+\frac{z}{r+\beta}\right]d\upsilon=\\&=L(t)\left[\frac{\eta}{\alpha+\eta}\Omega_0(t)\mathrm{e}^{-(\alpha+\eta)\pi}-\frac{z}{r+\beta}\mathrm{e}^{-\eta\pi}\right].\end{aligned} \tag{A1.10}$$

$$\begin{aligned}&\int_{t-\pi}^{t}L(\upsilon,t)\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\le\pi}d\upsilon=\\&=\int_{t-\pi}^{t}\eta\,\mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}\left[\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t)-\frac{t_W}{r+\beta}\left(1-\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right)+\frac{z}{r+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right]d\upsilon=\\&=L(t)\left[\frac{\eta}{\alpha+\eta}\Omega_0(t)\left(1-\mathrm{e}^{-(\alpha+\eta)\pi}\right)-\frac{t_W}{r+\beta}(1-\mathrm{e}^{-\eta\beta})+\frac{\eta(t_W+z)}{r+\beta}e^{-\beta\pi}\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right)\right].\end{aligned} \tag{A1.11}$$

Thus aggregate human wealth is:

$$A^H(t)=L(t)\left[\frac{\lambda\Omega_0(t)}{\alpha+\eta}+\eta\mathrm{e}^{-\beta\pi}\frac{t_W+z}{r+\beta}\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right)\right], \tag{A1.12}$$

where it was used that:

$$t_W(1-\mathrm{e}^{-\eta\pi})=z\mathrm{e}^{-\eta\pi}-d(t)\Rightarrow t_W+z=\frac{z-d(t)}{1-\mathrm{e}^{-\eta\pi}}.$$

From the expression for working-age households, taking into account $t_W=d+(t_W+z)\mathrm{e}^{-\eta\pi}$:

$$\begin{aligned}\bar{a}^H(t,t)&=\Omega_0(t)+\left(\frac{t_W+z}{r(t)+\beta}\right)\left(\mathrm{e}^{-(r(t)+\beta)\pi}-\mathrm{e}^{-\eta\pi}\right)-\frac{t_W}{r(t)+\beta}=\\&=\Omega_0(t)+\mathrm{e}^{-\beta\pi}\left(\frac{t_W+z}{r(t)+\beta}\right)\left(\mathrm{e}^{-r(t)\pi}-\mathrm{e}^{-n_L\pi}\right)-\frac{d(t)}{r(t)+\beta}.\end{aligned} \tag{A1.13}$$

After substituting this expression in the equation for A^H and eliminating $\Omega_0(t)$ we get:

$$\eta L(t)\bar{a}^H(t,t) = (\alpha+\eta)A^H(t) - \eta\gamma L(t), \tag{A1.14}$$

where $\gamma = \frac{-d(t)}{r+\beta} + \left(r+\alpha+\beta\right)\left(\frac{\mathrm{e}^{-\beta\pi}}{1-\mathrm{e}^{-\eta\pi}}\right)\left(\frac{z-d(t)}{r+\beta}\right)\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right).$

Taking into account the expression for $\bar{a}^H(t,t)$ and taking into account (19) we get:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \alpha + n^L - (\rho+\beta)\frac{\eta\gamma L(t) + (\alpha+\eta)A(t)}{C(t)}. \tag{A1.15}$$

2. Derivation of aggregate non-interest income

Aggregate non-interest income is defined as follows:

$$WI(t) = \int_{-\infty}^{t} L(\upsilon,t)WI(\upsilon,t)d\upsilon, \tag{A1.16}$$

where $WI(\upsilon,t)$ is defined as follows:

$$WI(\upsilon,\tau) = \begin{cases} (1-t_L)W^N(\upsilon,\tau) - t_W & \text{for } \tau-\upsilon \le \pi, \\ (1-t_L)W^N(\upsilon,\tau) + z & \text{for } \tau-\upsilon > \pi. \end{cases} \tag{A1.17}$$

Thus $WI(t)$ is split into parts, non-interest income of the retirees and non-interest income of the young:

$$WI(t) = \int_{-\infty}^{t-\pi} L(\upsilon,t)\left[(1-t_L)WI(\upsilon,t) + z\right]d\upsilon + \int_{t-\pi}^{t} L(\upsilon,t)\left[(1-t_L)WI(\upsilon,t) - t_W\right]d\upsilon. \tag{A1.18}$$

After applying the expression for $W^N(\upsilon,t) = E(t-\upsilon)F_N(k_N(t,1)) = \omega_0\mathrm{e}^{-\alpha(t-\upsilon)}F_N(k_N(t,1))$, which comes from taking into account the definition of the efficiency index $E(\tau-\upsilon)$ and the fact that the wage of a particular worker born at υ is equal to the marginal product of labor, adjusted for his or her productivity.

$$\begin{aligned} WI(t) = &\int_{-\infty}^{t-\pi} L(\upsilon,t)\left[(1-t_L)\omega_0\mathrm{e}^{-\alpha(t-\upsilon)}F_N(k_N(t,1)) + z\right]d\upsilon + \\ &+ \int_{t-\pi}^{t} L(\upsilon,t)\left[(1-t_L)\omega_0\mathrm{e}^{-\alpha(t-\upsilon)}F_N(k_N(t,1)) - t_W\right]d\upsilon. \end{aligned} \tag{A1.19}$$

Rearranging we get:

$$WI(t)=(1-t_L)F_N(k_N(t,1))\left[\int_{-\infty}^{t-\pi}L(\upsilon,t)\omega_0\mathrm{e}^{-\alpha(t-\upsilon)}d\upsilon+\int_{t-\pi}^{t}L(\upsilon,t)\omega_0\mathrm{e}^{-\alpha(t-\upsilon)}d\upsilon\right]+$$

$$+z\int_{-\infty}^{t-\pi}L(\upsilon,t)d\upsilon-t_W\int_{t-\pi}^{t}L(\upsilon,t)d\upsilon=(1-t_L)F_N(k_N(t,1))\int_{-\infty}^{t}L(\upsilon,t)\omega_0\mathrm{e}^{-\alpha(t-\upsilon)}d\upsilon+ \tag{A1.20}$$

$$+z\int_{-\infty}^{t-\pi}L(\upsilon,t)d\upsilon-t_W\int_{t-\pi}^{t}L(\upsilon,t)d\upsilon.$$

Noting from (15) that $\bar{n}(\upsilon,t)=E(t-\upsilon)=\omega_0\mathrm{e}^{-\alpha(t-\upsilon)}$ and applying the notion of $N(t)$ from (24) we get:

$$WI(t)=(1-t_L)F_N(k_N(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t)+z\int_{-\infty}^{t-\pi}L(\upsilon,t)d\upsilon-t_W\int_{t-\pi}^{t}L(\upsilon,t)d\upsilon. \tag{A1.21}$$

Applying (14) we get:

$$\begin{aligned}WI(t)&=(1-t_L)F_N(k_N(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t)+z\int_{-\infty}^{t-\pi}\eta\mathrm{e}^{\eta\upsilon-\beta t}d\upsilon-t_W\int_{t-\pi}^{t}\eta\mathrm{e}^{\eta\upsilon-\beta t}d\upsilon=\\&=(1-t_L)F_N(k(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t)-(1-\mathrm{e}^{-\eta\pi})t_W\mathrm{e}^{n_Lt}+z\mathrm{e}^{-\eta\pi}\mathrm{e}^{n_Lt}.\end{aligned} \tag{A1.22}$$

Taking into account the fact that the pension system is run on a non-balanced manner, thus using (34) the equation of the aggregate income can be simplified as follows:

$$WI(t)=(1-t_L)\frac{\eta\omega_0}{\alpha+\eta}F_N(k_N(t,1))L(t)+D(t). \tag{A1.23}$$

3. Social welfare

$$SW=\int_{t}^{\infty}\int_{-\infty}^{\tau-\pi}L(\upsilon,\tau)[ln\bar{c}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau+\int_{t}^{\infty}\int_{\tau-\pi}^{\tau}L(\upsilon,\tau)[ln\bar{c}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau. \tag{A1.24}$$

Taking into account the Euler equation:

$$\bar{c}(\upsilon,\tau)=\bar{c}(\upsilon,t)\mathrm{e}^{(r-\rho)(\tau-t)}, \tag{A1.25}$$

$$\bar{c}(\upsilon,\upsilon)=\bar{c}(\upsilon,t)\mathrm{e}^{(r-\rho)(\upsilon-t)}, \tag{A1.26}$$

$$(\rho+\beta)[\bar{a}(\upsilon,\upsilon)+\bar{a}^H(\upsilon,\upsilon)]=(\rho+\beta)[\bar{a}(\upsilon,t)+\bar{a}^H(\upsilon,t)]\mathrm{e}^{-(r(t)-\rho)(\upsilon-t)}. \tag{A1.27}$$

Applying that $\bar{a}(\upsilon,\upsilon)=0$ and simplifying we get the definition for the consumption of generation υ:

$$\bar{c}(\upsilon,\tau)=(\rho+\beta)(\bar{a}(\upsilon,\tau)+\bar{a}^H(\upsilon,\tau))=(\rho+\beta)\bar{a}^H(\upsilon,\upsilon)\mathrm{e}^{-(r-\rho)(\upsilon-t)}. \tag{A1.28}$$

$$\bar{a}_o^H(\upsilon,t) = \frac{1}{r+\alpha+\beta}\left(\omega(1-\varepsilon)(1-t_L)\left(\frac{k}{n}\right)^{\varepsilon} \mathrm{e}^{\alpha(\upsilon-t)}\right) + \frac{z}{r+\beta}, \tag{A1.29}$$

$$\bar{a}_y^H(\upsilon,t) = \frac{1}{r+\alpha+\beta}\left(\omega(1-\varepsilon)(1-t_L)\left(\frac{k}{n}\right)^{\varepsilon} \mathrm{e}^{\alpha(\upsilon-t)}\right) - \frac{tw}{r+\beta} + \frac{tw+z}{r+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}. \tag{A1.30}$$

Substituting the expressions of human wealth from above we get the following social welfare function as a function of the steady state level of capital per capita:

$$SW(t) = \frac{\mathrm{e}^{n_L t}}{n_L-\rho-\beta}\Big[(\ln((\rho+\beta)a_o^H) + (r-\rho)\frac{\pi\eta+1}{\eta})\mathrm{e}^{-\eta\pi} - \ln((\rho+\beta)a_y^H)(1-\mathrm{e}^{-\eta\pi}) - -(r-\rho)(1-\mathrm{e}^{\eta\pi} - \eta\pi\mathrm{e}^{-\eta\pi})\eta^{-1})\Big] \tag{A1.31}$$

4. Estimation results

Table A4.1. Steady states with the optimal choice of income tax rate ($\theta = 25\%$ **)**

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
t_L	38.0%	38.0%	38.0%	38.0%	38.0%	38.0%	38.0%	38.0%	38.0%
k	11.5	11.2	10.8	10.4	9.9	9.4	8.9	8.3	7.6
r	2.0%	2.1%	2.2%	2.4%	2.5%	2.7%	2.9%	3.2%	3.6%

Table A4.2 Optimal choice of income tax rate and social contributions ($\theta = 25\%$ **)**

	The share of non-Ricardian consumers λ								
	10%	*20%*	*30%*	*40%*	*50%*	*60%*	*70%*	*80%*	*90%*
t_L	43%	42%	42%	41%	41%	39%	38%	38%	37%
ψ	5%	6%	6%	6%	6%	48%	48%	48%	50%

Variables

$Y\,(y)$	output (per efficiency unit of labor)
$K^N\,(k^N)$	capital stock (per efficiency unit of labor)
$K\,(k)$	aggregate (per capita) capital stock
L	aggregate population size
$N\,(n)$	aggregate (per capita) labor supply in efficiency units of labor
W^N	nominal wage
W^K	nominal rental rate on capital
$\prod$	profit
r^N	real interest rate
$A\,(\bar{a},a)$	aggregate (individual, per capita) real financial wealth
$A^G\,(\bar{a}^G,a^G)$	aggregate (individual, per capita) government debt
$A_H^i\,(\bar{a}_H^i,a_H^i)$	aggregate (individual, per capita) real human wealth
$Z\,(\bar{z},z)$	nominal (real) transfer to retirees
$T_W\,(\bar{t}_W,t_W)$	nominal (real) tax on the young
$C^i\,(\bar{c}^i,c^i)$	aggregate (individual, per capita) consumption of Ricardian, non-Ricardian agents
$C^T\,(c^T)$	total aggregate (per capita) consumption
I	gross investment
$G\,(g)$	aggregate (per capita) government consumption
$D\,(d)$	deficit of the pension fund (per capita)
$c_y\,(d_y,a_y^G)$	ration of consumption (deficit of the pension fund, government debt) per capita to output per capita