The Role of Uncertain Government Preferences for Fiscal and Monetary Policy Interaction

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Abstract

This paper explores the role of uncertain government preferences in a linear-quadratic model of fiscal and monetary policy interaction. We show that the effects of preference uncertainty are fastened on multiplicative uncertainty about the policy effectiveness. If the effects of fiscal and monetary policies on the economy are known, preference uncertainty does not affect the symbiosis result of interaction. In this case, inflation and output are equal to their targets irrespective of the central bank and the government preferences. Multiplicative uncertainty about the fiscal policy effects creates the inflation bias, and preference uncertainty deteriorates it by lowering output and rising inflation up. Multiplicative uncertainty about the monetary policy effects creates either standard inflation bias or negative inflation bias with output higher than the target and inflation lower than the target. In this case, preference uncertainty enlarges the absolute value of the output gap, while the effect on the inflation gap depends on the extent of monetary multiplicative uncertainty. Thus, under some circumstances uncertain government preferences can even reduce the negative effect of multiplicative uncertainty. If the effects of both policies are uncertain, the impact of preference uncertainty depends not only on the extent of multiplicative uncertainty, but also on the inflation and output targets. After studying the impact of uncertainty on inflation and output gaps, we proceed with the welfare properties of the equilibrium and discuss the optimal conservativeness of authorities for different types of uncertainty.

Keywords: *fiscal and monetary policy interaction, multiplicative uncertainty, uncertain preferences*

JEL Codes: *E52, E58, E62, E63*

1 Introduction

Trump's inauguration has provoked the extensive debates among economists about the future fiscal policy stance in the U.S. Many analysts worry about the macroeconomic effects of this "Trump's uncertainty". It is too early to estimate its real economic effects, but it is already obvious, that the Fed's policy may be changed in response to this uncertainty. Some hint of possible changes can be found, for example, in the speech of the Fed Governor Lael Brainard on January 17, 2017 (Brainard et al. (2017)):

"There are many sources of uncertainty affecting... the appropriate path of monetary policy. In particular, there has been speculation about significant changes to fiscal policy of late, although the magnitude, composition, and timing of any fiscal changes are as yet unknown and will depend on the incoming Administration and the new Congress as well as the vicissitudes of the budgeting process... It thus seems possible that monetary policy could be affected for some time by uncertainty surrounding fiscal policy and its effects on the economy".

Starting from the famous paper by Sargent and Wallace (1981), fiscal and monetary policy interaction has been always in the center of attention in academic literature. One of the most important issues in this literature is whether the central bank and the government can achieve the target values of output and inflation. Up to the moment, there has been no consensus in this question.

Dixit and Lambertini (2003*b*) show that fiscal and monetary policy do achieve the target values of output and inflation if the government and the central bank share their targets. This result holds for all the forms of policy interaction and for all the weights in the loss functions. This conclusion is known as the symbiosis result. However, Dixit and Lambertini (2003*a*) show that if fiscal policy creates dead-weight loss and the targets of the central bank and the government are different, the non-cooperative equilibrium is characterized by inflation bias. This inflation bias with inflation higher than the target and output lower than the target arises because of too restrictive fiscal policy and too expansionary monetary policy.

Two papers by Di Bartolomeo et al. show that the symbiosis result also does not hold in case of multiplicative uncertainty. Di Bartolomeo, Giuli and Manzo (2009) investigate central bank and government interaction under multiplicative uncertainty about the fiscal policy effectiveness. They show that even if the government and the central bank share output and inflation target levels, fiscal multiplicative uncertainty does not allow them to achieve these targets. This uncertainty forces the government to become more cautious. As a result, fiscal policy becomes less expansionary and output drops. The central bank faces time inconsistency problem and tries to raise output with too expansionary policy, which leads to an increase in inflation, and the inflation bias arises. Di Bartolomeo and Giuli (2011) analyze multiplicative uncertainty about monetary policy effectiveness and come to the same result: multiplicative uncertainty causes ineffective levels of output and inflation in equilibrium. In their model, monetary multiplicative uncertainty forces the monetary authority to lower the absolute value of its intervention. This leads to the gap between the equilibrium inflation and its target. This effect could be neutralized by the change in fiscal policy, which can be done at sake of the gap between the equilibrium output and the target level. Obviously, the government is reluctant to change considerably the policy and none of the targets is achieved.

In our paper, we examine these results in the model with uncertain government preferences. We assume that the government knows its own preferences, while for the others the government preferences are uncertain. To our knowledge, there are no other studies of fiscal and monetary policy interaction with uncertain government preferences. The role of uncertain central bank preferences has been already studied in economic literature. Ciccarone, Marchetti and Di Bartolomeo (2007), Hefeker and Zimmer (2011) show that uncertainty about the central bank preferences could reduce the macroeconomic volatility due to the fiscal disciplining effect, which is expressed in reduction of taxes, inflation and output distortions. Dai and Sidiropoulos (2011), however, note that such result can be achieved only under the Stackelberg interaction, where the government acts as a leader and the central bank acts as a follower. Dai and Sidiropoulos (2011) argue that the fiscal disciplining effect of uncertain central bank preferences could be insignificant if the government and the central bank move simultaneously. Oros and Zimmer (2015) analyze the monetary transmission mechanism in a monetary union with uncertain central bank preferences. They show that the private agents expect the central bank to be more conservative to compensate the uncertainty of the central bank preferences. This could lead to a decrease in inflation and better macroeconomic outcomes not because of a disciplinary effect, but because of the central bank's communication channel.

Thus, as we have seen, economic literature elaborates a number of applications of uncertainty about the central bank preferences for strategic interaction between fiscal and monetary policy. However, the existing research has not been dealing with uncertainty about the government preferences. Meanwhile, uncertainty about the government preferences seems to be much more significant than uncertainty about the central bank preferences, at least in developed countries. For example, the targets of the European Central Bank are clearly defined: inflation below and close to 2 percent. Moreover, Blinder et al. (2008) show that in recent years transparency of monetary policy has considerably increased all over the world. This means that the assumption of uncertain central bank preferences might be unjustified. At the same time, taking into account uncertain government preferences seems to be promising. Firstly, the government preferences are exposed to considerable changes in the election period. Moreover, fiscal authorities have not been demonstrating considerable improvements in their information policies in recent years. Almost everywhere, the governments are much less transparent than the central banks.

The goal of our paper is to study the effects of uncertain government preferences on fiscal and monetary policy interaction. We show that uncertainty about the government preferences does not change the interaction result if the policy effects are certain. However, uncertain government preferences matter in case of multiplicative policy uncertainty. Below we show how uncertainty about the government preferences affects macroeconomic equilibrium under fiscal and/or monetary multiplicative uncertainty.

The paper is organized as follows. In Section I we describe a benchmark model of fiscal and monetary policy interaction. Section II analyzes the equilibrium in the model with certain preferences. In Section III we discuss the impact of uncertain government preferences on the equilibrium. Section IV concludes.

2 Benchmark Model

We start our analysis with a standard benchmark model with certain preferences from Dixit and Lambertini (2000, 2003*b*). This model is described by two equations: aggregated demand (1) and aggregated supply (2):

$$\pi = \varphi m + \rho c \tau \tag{1}$$

$$y = \overline{y} + b\left(\pi - \pi^e\right) + a\tau \tag{2}$$

where π is the rate of inflation, π^e is the expected rate of inflation, y is the level of real output, $\overline{y}$ is the natural level of real output, τ is the instrument of fiscal policy (for example, transfers), m is the monetary policy instrument (for example, the growth rate of the money supply). The effect of monetary policy on inflation is prone to a multiplicative shock φ with mean 1 and variance σ_φ^2. Parameter σ_φ^2 characterizes the degree of monetary multiplicative uncertainty. The average effect of fiscal policy on inflation is given by variable c. The fiscal effect on inflation is hit by multiplicative shock ρ with mean 1 and variance σ_ρ^2. Thus, parameter σ_ρ^2 characterizes the degree of fiscal multiplicative uncertainty. Parameter $b > 0$ characterizes the indirect effect of policies on the output through inflation surprise, while a is the direct effect of fiscal policy on output.

Dixit and Lambertini (2000) and complementary appendix to Dixit and Lambertini (2003*a*) show that equations (1) and (2) represent the log-linearization of equilibrium in a micro-founded general-equilibrium model. This model describes an economy inhabited by a number of individuals each of which produces a single good, sells it in a monopolistically competitive market and consumes a bundle of goods. The central bank in this economy controls money supply. An increase in money supply leads to an increase in aggregate demand and to an increase in inflation. The government in the economy may set taxes, transfers and government spendings under constraint of balanced budget. Different fiscal policy regimes implies different signs of coefficient a and c. For example, Dixit and Lambertini (2003a) assume that government sets a proportional subsidy on sales and lump-sum taxes to balance the budget. In this case an increase in proportional subsidy leads to an increase in output and to a decrease in inflation rate, meaning that a is positive and c is negative. Dixit and Lambertini (2000) mention the case of distortionary taxes and wasting government spendings. A decrease in tax rate leads to an increase in both inflation and output. This implies that both a and c are positive, if τ is treated as the opposite to tax rate. Moreover, Dixit and Lambertini (2000) show that a is negative and c is positive, if income-tax revenues are spent on government spendings.

Thus, both a and c can be of either sign. For tractability reasons and to keep our results comparabale to Di Bartolomeo, Giuli and Manzo (2009) and Di Bartolomeo and Giuli (2011), we assume that $c > 0$ and $a > 0$. Nevertheless, all the algebra in the paper remains the same for other signs of the parameters.

Our model generalizes two papers: Di Bartolomeo, Giuli and Manzo (2009), which studies fiscal multiplicative uncertainty, and Di Bartolomeo and Giuli (2011), which studies monetary multiplicative uncertainty. The results of both papers can be easily replicated in our model by putting the corresponding variance to zero. Moreover, our model allow us to study the additional effects which arise only if both multiplicative shocks are present.

Losses of the central bank and the government are defined by the gap between inflation rate and the target inflation π^* and by the gap between output and the target output y^*:

$$L_{CB} = E\left[(\pi - \pi^*)^2 + \theta_B (y - y^*)^2\right] \tag{3}$$

$$L_G = E\left[(\pi - \pi^*)^2 + \theta_G (y - y^*)^2\right] \tag{4}$$

$$\theta_B > 0, \theta_G > 0,$$

where θ_B and θ_G characterize the preferences of the central bank and the government for output. To stay in line with the broad consensus in the literature (see, for example, Rogoff (1985)), we assume that the central bank is more conservative than the government: $\theta_G \geq \theta_B$. Moreover, the output target is higher than the natural level: $y^* > \bar{y}$. In our model, the government and the central bank choose their policies simultaneously and independently after the expectations have been formed. Minimization of losses (3) and (4) subject to constraints (1) and (2) gives the following reaction functions:

$$\tau(\theta_G) = \frac{-c\left(\overline{m}-\pi^*\right)+\theta_G\left(a+bc\right)\left(y^*-\overline{y}+b\pi^e-b\overline{m}\right)}{c^2\left(1+\sigma_\rho^2\right)+\theta_G\left(\sigma_\rho^2 b^2 c^2+\left(a+bc\right)^2\right)} \tag{5}$$

$$m(\theta_B) = \frac{\pi^*-c\overline{\tau}+b\theta_B\left(y^*-\overline{y}+b\pi^e-\left(a+bc\right)\overline{\tau}\right)}{\left(1+\sigma_\varphi^2\right)\left(1+\theta_B b^2\right)}, \tag{6}$$

where (5) is the reaction function of the government with preferences θ_G, (6) is the reaction function of the central bank with preferences θ_B, $\overline{m}$ is the expected value of monetary instrument and $\overline{\tau}$ is the expected value of fiscal instrument. As we can see from (5) and (6), the equilibrium values of both policy instruments depend positively on the inflation target π^*, expected inflation π^e and the gap between target and natural output $(y^*-\overline{y})$. The impact of the output gap on a policy instrument depends positively on the weight of output in a policymaker's loss function. According to (6), the absolute value of monetary instrument chosen by the central bank depends negatively on the variance of monetary multiplicative shock σ_φ^2. This phenomenon corresponds to the standard attenuation effect, explored by **?**: uncertainty about the policy instrument forces the policymaker to become more cautious and to decrease the extent of intervention. The same attenuation effect is true for the government. According to (5), the absolute value of fiscal instrument τ decreases with the extent of fiscal multiplicative uncertainty, measured by σ_ρ^2.

3 Equilibrium with certain preferences

In this Section we look for the equilibrium with certain preferences. We assume that the parameter of monetary preferences θ_B is equal to $\tilde{\theta}_B$ and the parameter of the government preferences θ_G is equal to $\tilde{\theta}_G$. As the preferences of both policymakers are known by all the agents, the expected values of their policy instruments coincide with their actual values: $\overline{m}=m(\tilde{\theta}_B)$ and $\overline{\tau}=\tau(\tilde{\theta}_G)$.

We start with the equilibrium with certain policy effects, which corresponds to the model of Dixit and Lambertini (2003*b*). Substituting $\sigma_\rho^2=0$, $\sigma_\varphi^2=0$ into reaction functions (5) and (6), we obtain the following equilibrium values of fiscal and monetary instruments:

$$\tau_0 = \frac{y^*-\overline{y}}{a} \tag{7}$$

$$m_0 = \pi^* - c\tau_0 \tag{8}$$

As the target output is higher than the natural level, in equilibrium the fiscal policy is expansionary: $\tau_0>0$. The value of the fiscal instrument (7) is chosen in a such way that the equilibrium level of output coincides with the target value: $y=y^*$. Expansionary fiscal policy would lead to an increase in the inflation rate, equal to $c\tau_0$. Nevertheless, the central bank can react to this inflationary pressure by decreasing the monetary instrument by the same value. The sign of equilibrium value of m_0 depends on the value of inflation target. If inflation target is sufficiently high, such that $\pi^* > \frac{c}{a}(y^* - \overline{y})$, monetary policy is expansionary and $m_0 > 0$. If inflation target is low, the equilibrium monetary policy is contractionary, $m_0 < 0$. As a result, the equilibrium inflation rate is equal to the target: $\pi = \pi^*$. Thus, the model with certain policy effects replicates the symbiosis result of Dixit and Lambertini (2003*b*): irrespective of their preferences, the central bank and the government achieve their inflation and output targets.

If both the policy effects are uncertain, the intersection of (5) and (6) for given $\tilde{\theta}_G$ and $\tilde{\theta}_B$ brings the following equilibrium values of fiscal and monetary instruments:

$$\tilde{\tau} = \tau_0 - \frac{\tilde{W}_\tau}{\tilde{W}}\tau_0 - \frac{\tilde{W}_\tau \Lambda_B}{\tilde{W}}\tau_0 + \frac{\tilde{W}_m - \Lambda_B \tilde{\theta}_G a\,(a+bc)}{\tilde{W}}\frac{m_0}{c} \tag{9}$$

$$\tilde{m} = m_0 - \frac{\tilde{W}_m}{\tilde{W}}m_0 - \frac{\tilde{W}_\tau \Lambda_B}{\tilde{W}}m_0 + \left(c + ab\tilde{\theta}_B\right)\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0, \tag{10}$$

where $\Lambda_G = \sigma_\rho^2\left(\tilde{\theta}_G b^2 + 1\right)$, $\Lambda_B = \sigma_\varphi^2\left(\tilde{\theta}_B b^2 + 1\right)$, $\tilde{W}_\tau = c^2\Lambda_G$, $\tilde{W}_m = \Lambda_B\left(c^2 + \tilde{\theta}_G a\,(a+bc)\right)$, $\tilde{W} = W + \tilde{W}_\tau + \tilde{W}_m + \Lambda_G\Lambda_B c^2$ and $W = a\left(\tilde{\theta}_G a + \left(\tilde{\theta}_G - \tilde{\theta}_B\right)bc\right)$.

According to (9) and (10), the equilibrium values of policy instruments $\tilde{\tau}$ and $\tilde{m}$ are affected by multiplicative uncertainty. We can distinguish three effects: the direct effect of fiscal multiplicative uncertainty, the direct effect of monetary multiplicative uncertainty and the mutual effect which arises only if both uncertainties are present.

The direct effect of fiscal multiplicative uncertainty corresponds qualitatively to the process described in Di Bartolomeo, Giuli and Manzo (2009). Fiscal multiplicative uncertainty forces the government to attenuate its policy and to decrease τ. This attenuation effect is equal to $\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0$ and depends positively on the uncertainty extent σ_ρ^2. Moreover, the size of the attenuation effect depends negatively on $\tilde{\theta}_G$. More the government prefers output, less is the decrease in τ in response to uncertainty. The fiscal attenuation leads to a decrease in both output and inflation, which drop lower than their desired levels. In response to a decrease in τ, the central bank starts to stimulate economy with a more expansionary policy. An increase in monetary instrument equal to $c\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0$ would be enough to compensate the drop in inflation rate due to the attenuation effect of fiscal policy. Nevertheless, similarly to the famous paper Kydland and Prescott (1977), an inflation bias arises. The central bank takes inflation expectations as given and tries to push output up. With this goal, the central bank raises monetary instrument more than necessary to stabilize inflation.

As we can see from (10), the excess response of monetary policy to fiscal multiplicative uncertainty is equal to $ab\tilde{\theta}_B\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0$. This excess increase in monetary instrument depends positively on the monetary preferences of output, $\tilde{\theta}_B$. Due to this excess increase in monetary instrument, expected inflation becomes higher than the optimal level. This, nevertheless, cannot overcome the output drop caused by the decrease in fiscal instrument, as only fiscal policy can affect the output in equilibrium.

Thus, the direct effect of fiscal multiplicative uncertainty is the inflation bias, which corresponds to the Di Bartolomeo, Giuli and Manzo (2009). Nevertheless, as the ratio $\frac{\tilde{W}_\tau}{\tilde{W}}$ depends negatively on the variance of monetary multiplicative shock, σ_φ^2, we can conclude that the presence of monetary uncertainty decreases the inflation pressure of fiscal attenuation. The intuition is straightforward: as the central bank is unsure about the monetary policy effectiveness, monetary policy also becomes more cautious. Thus, the central bank allows a lower excess increase in monetary instrument and the increase in inflation is lower.

The direct effect of monetary multiplicative uncertainty on monetary policy is equal to $-\frac{\tilde{W}_m}{\tilde{W}} m_0$ and corresponds qualitatively to the effect described in Di Bartolomeo and Giuli (2011). Uncertainty about the monetary policy effectiveness leads to the attenuation effect in monetary policy and the absolute value of monetary instrument drops. The government reacts to the attenuation effect in monetary policy by the opposite change in fiscal instrument. The change in τ equal to $\frac{\tilde{W}_m}{\tilde{W}} \frac{m_0}{c}$ would be enough to overcome the effect on inflation. Nevertheless, this would influence the output and the government varies fiscal instrument less. The change in τ is proportional to $\frac{\tilde{W}_m - \Lambda_B \tilde{\theta}_G a (a + bc)}{\tilde{W}}$. The stronger preferences for output $\tilde{\theta}_G$, the less change in fiscal instrument.

The influence of monetary multiplicative uncertainty on expected output and inflation depends on the sign of m_0. If $m_0 > 0$, monetary multiplicative uncertainty forces the central bank to decrease m and monetary policy becomes more contractionary. The government responds to this by an increase in fiscal instrument. This, in turn, leads to an increase in output. In order to prevent output from the excess increase, the government raises its instrument to a less extent than is necessary to overreact the influence on inflation. Moreover, the equilibrium fiscal instrument decreases with $\tilde{\theta}_G$. As a result, a negative inflation bias arises with expected inflation less than π^* and expected output greater than y^*.

On the contrary, if $m_0 < 0$, monetary multiplicative uncertainty makes monetary policy more expansionary. The government reacts by a decrease in τ. This decrease is less than necessary to overreact inflationary impact of monetary policy. As a result, expected inflation is higher than π^*, while output is lower than y^*. In other words, inflation bias arises.

As we already noted, the direct effects of fiscal and monetary uncertainties correspond qualitatively to the conclusions of Di Bartolomeo, Giuli and Manzo (2009) and Di Bartolomeo and Giuli (2011). Nevertheless, the simultaneous presence of both sources of uncertainty creates some additional effects. These effects are proportional to the product of Λ_G and Λ_B in equations

(9) and (10). First of all, simultaneous uncertainty about both policies decreases the response of any policymaker to the uncertainty about the other's policy effectiveness. This follows directly from (9) and (10) if we remember that $\tilde{W}$ depends positively on the product $\Lambda_B\Lambda_G$. On the other hand, the mutual uncertainty influences the direct effects of both sources. For example, the presence of monetary uncertainty aggravates the attenuation effect which is caused by fiscal uncertainty. Fiscal instrument drops by additional amount of $\frac{c^2\Lambda_G\Lambda_B}{\tilde{W}}\tau_0$. Moreover, this decrease is not compensated by an increase in a monetary instrument. Thus, the mutual effect strengthens the negative effect of fiscal uncertainty on the output and weakens the upward shift in inflation. The mutual effect also strengthens the attenuation in monetary policy by the amount of $\frac{c^2\Lambda_G\Lambda_B}{\tilde{W}}$. This change in monetary instrument is not compensated by a corresponding response of fiscal authority. Thus, the mutual uncertainty weakens the effect of monetary uncertainty on inflation.

The overall effect of uncertainty on the equilibrium depends on the comparative strength of all these effects. The expected levels of output and inflation can be obtained from (1), (2) together with (9), (10) and are as follows:

$$\tilde{\pi}^e = \pi^*\left(1 - \frac{c^2\Lambda_G\Lambda_B}{\tilde{W}}\right) + \frac{a\tilde{\theta}_B b\tilde{W}_\tau}{\tilde{W}}\tau_0 - \frac{\Lambda_B\tilde{\theta}_G a\,(a+bc)}{\tilde{W}}m_0 \tag{11}$$

$$\tilde{y}^e = y^* + \frac{ac^2\Lambda_B}{\tilde{W}}\frac{m_0}{c} - \frac{a\tilde{W}_\tau\,(1+\Lambda_B)}{\tilde{W}}\tau_0 \tag{12}$$

According to (11), the gap between expected inflation and its target depends on the direct effects of multiplicative uncertainty and the mutual effect described above. The direct effect of fiscal uncertainty is equal to $\frac{a\tilde{\theta}_B b\tilde{W}_\tau}{\tilde{W}}\tau_0$. This effect is explained by the overreaction of the central bank to the attenuation in fiscal policy. The underreaction of the government to the attenuation in monetary policy leads to the change in inflation equal to $-\frac{\Lambda_B\tilde{\theta}_G a(a+bc)}{\tilde{W}}m_0$. As we discussed earlier, this effect is positive if m_0 is negative and vice versa. The coexistence of both sources of uncertainty leads to the additional attenuation of the policies. This forces a further decrease in inflation, equal to $\frac{c^2\Lambda_G\Lambda_B}{\tilde{W}}\pi^*$.

The attenuation effect of fiscal policy leads to a decrease in the output, equal to $\frac{a\tilde{W}_\tau}{\tilde{W}}\tau_0$. The presence of monetary multiplicative uncertainty strengthens this attenuation effect and causes a further decrease in output, equal to $\frac{a\tilde{W}_\tau\Lambda_B}{\tilde{W}}\tau_0$. The under-reaction of the government to the attenuation in monetary policy leads to the change in output equal to $\frac{ac^2\Lambda_B}{\tilde{W}}\frac{m_0}{c}$. This amount is positive if m_0 is positive. If m_0 is negative, all the effects of uncertainty on output are negative.

The general properties of the equilibrium are summarized by Proposition 2.1:

Proposition 1. *For given* $(\tilde{\theta}_B,\ \tilde{\theta}_G,\ \sigma_\rho^2,\ \sigma_\varphi^2)$*, there exist* $\lambda_2 \geq \lambda_1$*, such that in equilibrium with certain preferences:*

i) $\pi^e \geq \pi^*$ *if and only if* $\dfrac{m_0}{\tau_0} \leq \lambda_1$*;*

ii) $y^e \geq y^*$ *if and only if* $\dfrac{m_0}{\tau_0} \geq \lambda_2$*;*

where $\lambda_1 = \dfrac{c^2\Lambda_G\left(ab\tilde{\theta}_B - c\Lambda_B\right)}{\Lambda_B\left(c^2\Lambda_G + \tilde{\theta}_G a\,(a+bc)\right)}$, $\lambda_2 = \dfrac{c\Lambda_G\,(1+\Lambda_B)}{\Lambda_B} \geq 0.$

Proof. See Eqs. (11) and (12). □

Proposition 2.1 indicates that there can be three different economic situations in equilibrium. If $\dfrac{m_0}{\tau_0} \leq \lambda_1$, there is an inflation bias problem: the expected rate of inflation exceeds its target level $(\pi^e \geq \pi^*)$, while the expected rate of output is below its target level $(y^e \leq y^*)$. If $\lambda_1 < \dfrac{m_0}{\tau_0} \leq \lambda_2$, there is the deflation bias problem: both the expected rate of inflation and output are below their target levels $(\pi^e \leq \pi^*, y^e \leq y^*)$. If $\dfrac{m_0}{\tau_0} > \lambda_2$, there is a negative inflation bias problem: the expected rate of output exceeds its target level $(y^e \geq y^*)$, while the expected level of inflation is below its target level $(\pi^e \leq \pi^*)$.

We can also note that if we set $\sigma_\varphi^2 = 0$, we automatically replicate the results of Di Bartolomeo, Giuli and Manzo (2009). In this case both the thresholds λ_1 and λ_2 go to infinity and for any possible $\dfrac{m_0}{\tau_0}$ the economy faces the inflation bias problem. If σ_ρ^2 increases, the inflation bias problem aggravates.

If we let $\sigma_\rho^2 = 0$, we get the result of Di Bartolomeo and Giuli (2011). In this case, both the thresholds are equal to zero. This means that if $\dfrac{m_0}{\tau_0} < 0$, there is the inflation bias problem in the economy. If $\dfrac{m_0}{\tau_0} > 0$, there is negative inflation bias.

The simultaneous presence of monetary and fiscal multiplicative uncertainty makes the third type of equilibrium possible. This equilibrium is characterized by both inflation and output lower than their targets and is achieved at intermediate values of $\dfrac{m_0}{\tau_0} \in (\lambda_1, \lambda_2)$. It is easy to show that $\dfrac{\partial\lambda_1}{\partial\sigma_\rho^2} > 0$, $\dfrac{\partial\lambda_1}{\partial\sigma_\phi^2} < 0$, $\dfrac{\partial\lambda_2}{\partial\sigma_\rho^2} > 0$ and $\dfrac{\partial\lambda_2}{\partial\sigma_\phi^2} < 0$. Moreover, λ_1 is positive if and only if $\sigma_\phi^2 > \dfrac{ab\tilde{\theta}_B}{1+b^2\tilde{\theta}_B^2}$, while λ_2 is always positive. After characterizing the equilibrium with certain preferences, we now proceed to the search for the equilibrium with preference uncertainty.

4 Uncertain government preferences

In this Section, we relax the assumption of certain preferences and assume that parameter θ_G is a random variable with mean $\tilde{\theta}_G$ and cumulative distribution function $F(\theta_G)$ with support $[\underline{\theta}_G, \overline{\theta}_G]$. Thus, we can rewrite the reaction function of the government with preferences θ_G (5) in the following way:

$$\tau\left(\theta_G\right) = \tau\left(\tilde{\theta}_G\right) - \Phi_G \omega\left(\theta_G\right), \tag{13}$$

where $\tau\left(\tilde{\theta}_G\right)$ is the value of fiscal instrument chosen by the government with preferences $\tilde{\theta}_G$, $\Phi_G = \dfrac{c^2\left(1+\sigma_\rho^2\right)(a+bc)\left(y^* - \overline{y} + b\pi^e - c\pi^*\right) + ac\left(a + bc\left(1-\sigma_\rho^2\right)\right)(m-\pi^*)}{c^2\left(1+\sigma_\rho^2\right) + \tilde{\theta}_G\left(\sigma_\rho^2 b^2 c^2 + (a+bc)^2\right)}$ and $\omega(\theta_G) = \dfrac{\tilde{\theta}_G - \theta_G}{c^2\left(1+\sigma_\rho^2\right) + \theta_G\left(\sigma_\rho^2 b^2 c^2 + (a+bc)^2\right)}$ characterizes the distance between the actual government preferences θ_G and the mean preferences $\tilde{\theta}_G$, with $\dfrac{\partial\omega}{\partial\theta_G} < 0$ and $\dfrac{\partial^2\omega}{\left(\partial\theta_G\right)^2} > 0$.

The central bank does not know the true distance between the government preferences and their mean, so the monetary policy is conducted according to equation (6), which is the reaction of the central bank to the expected value of fiscal instrument, $\overline{\tau}$. The expected value of fiscal instrument can be computed with the help of (13):

$$\overline{\tau} = \tau\left(\tilde{\theta}_G\right) - \Phi_G \Omega_G, \tag{14}$$

where $\Omega_G = \int_{\underline{\theta_G}}^{\overline{\theta_G}} \omega\left(\theta_G\right) \mathrm{d}F\left(\theta_G\right)$ is the average value of $\omega\left(\theta_G\right)$. As function $\omega\left(\theta_G\right)$ is decreasing and convex, Ω_G is higher than the value $\omega\left(\tilde{\theta}_G\right)$, which is equal to zero. Obviously, the value of Ω_G depends on the extent of uncertainty about the government preferences. Due to convexity of function $\omega\left(\theta_G\right)$, the higher variance of θ_G the higher value of Ω_G.

To compute the equilibrium, we firstly find the intersection of reaction functions (6) and (14). After that, we compute expected inflation in the intersection point and substitute it into the reaction functions. The equilibrium values of policy instruments are as follows:

$$\hat{\overline{\tau}} = \tau_0 - \frac{\hat{W}_\tau\left(1+\Lambda_B\right)}{\hat{W}}\tau_0 + \frac{\hat{W}_m - \Lambda_B a\left(a+bc\right)\tilde{\theta}_G - ac^2\Omega_G\Lambda_B\left(bc\sigma_\rho^2 - (a+bc)\right)}{\hat{W}}\frac{m_0}{c} \tag{15}$$

$$\hat{\tau}\left(\theta_G\right) = \hat{\overline{\tau}} + \left(-\frac{\alpha_\tau}{\hat{W}}\tau_0 + \frac{\alpha_m}{\hat{W}}\frac{m_0}{c}\right)\left(\omega\left(\theta_G\right) - \Omega_G\right) \tag{16}$$

$$\hat{m} = m_0 - \frac{\hat{W}_m + c^2\Lambda_B\Lambda_G}{\hat{W}}m_0 + \left(c + ab\tilde{\theta}_B\right)\frac{\hat{W}_\tau}{\hat{W}}\tau_0, \tag{17}$$

where (15) is the average fiscal policy in equilibrium, (16) is the equilibrium policy of a government with preferences θ_G, (17) is the equilibrium monetary policy, $\hat{W}_\tau = \tilde{W}_\tau + \Omega_G\sigma_\rho^2 c^2 a\,(a+2bc)$, $\hat{W}_m = \tilde{W}_m + \Lambda_B bc^3\,(a+bc)\,\Omega_G(1+\sigma_\rho^2)$, $\hat{W} = \tilde{W} - \Omega_G c\left(a\,(a+bc)\left(b\,(a+bc)\,\tilde{\theta}_B + c\right) + \sigma_\rho^2 abc^2\left(b^2\tilde{\theta}_B - 1\right)\right) - bc^3\,(a+bc)\,\Lambda_B\left(1+\sigma_\rho^2\right)\Omega_G$, $\alpha_m = c^2\Lambda_B\left(a\,(a+bc) + \sigma_\rho^2 b^2c^2\right)$, $\alpha_\tau = \sigma_\rho^2 c^2\left[a\,(a+bc) + \tilde{\theta}_B ab^3c + \Lambda_B\left(a\,(a+bc) - b^2c^2\right)\right]$.

If we compare (15) and (17) with the equilibrium policies with certain preferences (9) and (10), we will see that the main effects created by uncertainty are the same. These are the fiscal attenuation effect equal to $-\frac{\hat{W}_\tau\,(1+\Lambda_B)}{\hat{W}}\tau_0$ in (15) and the monetary attenuation effect equal to $-\frac{\hat{W}_m + c^2\Lambda_B\Lambda_G}{\hat{W}}m_0$ in (17). The reaction of the central bank to the fiscal attenuation effect is given by $-\left(c + ab\tilde{\theta}_B\right)\frac{\hat{W}_\tau}{\hat{W}}\tau_0$ in (17), while the average reaction of fiscal policy to the monetary attenuation effect is given by $\frac{\hat{W}_m - \Lambda_B a\,(a+bc)\,\tilde{\theta}_G - ac^2\Omega_G\Lambda_B\left(bc\sigma_\rho^2 - (a+bc)\right)}{\hat{W}}\frac{m_0}{c}$ in (15). These effects define the expected inflation and output in equilibrium:

$$\begin{aligned}\hat{\pi}^e =& \pi^* + \frac{\Lambda_B\left(\left(c^2 + a\,(a+bc)\,\tilde{\theta}_G + c^2\Lambda_G\right) + bc^3\,(a+bc)\,\Omega_G\left(1+\sigma_\rho^2\right)\right)}{\hat{W}}m_0 + \\ &+ \frac{\hat{W}_\tau\left(ab\tilde{\theta}_B - c\Lambda_B\right)}{\hat{W}}\tau_0\end{aligned} \tag{18}$$

$$\hat{y}^e = y^* + \frac{ac^2\Lambda_B\left(1 + \Omega_G\left((a+bc)^2 + b^2c^2\sigma_\rho^2\right)\right)}{\hat{W}}\frac{m_o}{c} - \frac{a\hat{W}_\tau\,(1+\Lambda_B)}{\hat{W}}\tau_0 \tag{19}$$

As we can see, the equilibrium values of monetary and fiscal instruments are given by the cumbersome equations. Thus, we start the discussion of the equilibrium with the polar cases when either σ_ρ^2 or σ_φ^2 is equal to zero. After that, we describe the equilibrium in the generalized model with both σ_ρ^2 and σ_φ^2 positive.

4.1 Certain policy effects and uncertain fiscal preferences

We start to analyze the effects of preference uncertainty in the model with $\sigma_\rho^2 = \sigma_\varphi^2 = 0$:

Proposition 2. *In equilibrium with uncertain government preferences and without multiplicative uncertainty, $m = m_0$, $\tau(\theta_G) = \tau_0$ for any θ_G. Thus, for any Ω_G equilibrium output and inflation are equal to their target levels: $y = y^*$, $\pi = \pi^*$.*

Proof. Substitute $\sigma_\rho^2 = 0$ and $\sigma_\varphi^2 = 0$ into Eqs. (15-19). □

Proposition 2 indicates that in the absence of multiplicative uncertainty the government preference uncertainty does not affect the equilibrium. Irrespective of its preferences, the government with any θ_G chooses τ_0. Thus, the average fiscal policy is also equal to τ_0. The optimal reaction of the central bank to the average τ_0 is equal to m_0. As a result, in this case the uncertainty about the government preferences is not relevant and the symbiosis result of Dixit and Lambertini (2003*b*) holds: the government and the central bank are able to achieve both inflation and output targets.

4.2 Fiscal multiplicative uncertainty and fiscal preference uncertainty

We proceed with the model with fiscal multiplicative uncertainty. The equilibrium in this model is described in the following Proposition:

Proposition 3. *The equilibrium with fiscal multiplicative uncertainty and government preference uncertainty* $(\sigma_\rho^2 > 0, \Omega_G > 0, \sigma_\varphi^2 = 0)$ *is such that:*

i) *For any* $\frac{m_0}{\tau_0}$*, there is the inflation bias problem: the expected rate of inflation exceeds its target level* $(\pi^e > \pi^*)$*, while the expected rate of output is below its target level* $(y^e < y^*)$*.*

ii) *Government preferences uncertainty aggravates the inflation bias problem. With higher* Ω_G*, the inflation gap and the output gap become larger:* $\frac{\partial|\pi^e - \pi^*|}{\partial\Omega_G} > 0$, $\frac{\partial|y^e - y^*|}{\partial\Omega_G} > 0$.

Proof. Substitute $\sigma_\varphi^2 = 0$ into Eqs. (15-19). □

Part i) of Proposition 3 states that the equilibrium with fiscal multiplicative and preferences uncertainty is characterized by inflation bias. The intuition is straightforward. The fiscal multiplicative uncertainty leads to the attenuation fiscal effect. The central bank does not know the true preferences of the government and has to rely on the average fiscal attenuation effect, which is given by the term $\frac{\hat{W}_\tau}{\hat{W}}\tau_0$ in (15). The attenuation fiscal effect leads to a decrease in both inflation and output. An increase in monetary instrument equal to $c\frac{\hat{W}_\tau}{\hat{W}}\tau_0$ would be enough to compensate the average decrease in inflation due to fiscal multiplicative uncertainty. Nevertheless, the central bank takes expectations as given and raises its instrument more in order to stimulate output. The value of the excess increase in monetary instrument is proportional to $ab\tilde{\theta}_B\frac{\hat{W}_\tau}{\hat{W}}$. This excess increase in monetary instrument pushes inflation above the target level, while expected output stays below the target.

Part ii) of Proposition 3 states that an increase in the dispersion of fiscal preferences leads to the higher inflation bias. To understand this, note that the gap between expected output and the target is proportional to the average attenuation fiscal effect. From equation (14), the value of the average fiscal instrument $\overline{\tau}$ is lower than $\tau(\tilde{\theta}_G)$. Thus, the average attenuation effect is higher than the attenuation of the policy by the government with preferences $\tilde{\theta}_G$. With higher preference uncertainty, measured by Ω_G, the difference between the average attenuation and the attenuation of the government with average preferences becomes larger. Consequently, the absolute value of the expected output gap also increases. Thus, the willingness of the central bank to stimulate output with the excessive increase in monetary instrument enlarges. As a result, the gap between expected inflation and the target inflation becomes larger.

The effects of fiscal multiplicative uncertainty in the model with uncertain government preferences coincide with the effects in the model with certain preferences qualitatively and are larger quantitatively. In the next subsection we analyze the effects of preference uncertainty in the model with monetary multiplicative shocks.

4.3 Monetary multiplicative uncertainty and fiscal preference uncertainty

Now we proceed to the model with monetary multiplicative uncertainty. The equilibrium in this model is described in the following Proposition 2.4:

Proposition 4. *The equilibrium with monetary multiplicative uncertainty and government preference uncertainty ($\sigma_\rho^2 = 0, \Omega_G > 0, \sigma_\varphi^2 > 0$) is such that:*

i) *If $m_0 > 0$, there is negative inflation bias problem in the economy: the expected rate of output exceeds its target level ($y^e \geq y^*$), while the expected level of inflation is below its target level ($\pi^e \leq \pi^*$). If $m_0 < 0$, there is the inflation bias problem in the economy: the expected rate of inflation exceeds its target level ($\pi^e \geq \pi^*$), while the expected rate of output is below its target level ($y^e \leq y^*$).*

ii) $\dfrac{\partial|\pi^e - \pi^*|}{\partial\Omega_G} \geq (\leq) 0$ *if and only if* $\sigma_\varphi^2 \leq (\geq) \dfrac{ab\tilde{\theta}_B}{c\left(1 + b^2\tilde{\theta}_B\right)}$. *If* $\sigma_\varphi^2 > \dfrac{ab\tilde{\theta}_B}{c\left(1 + b^2\tilde{\theta}_B\right)}$, *an increase in Ω_G lowers the inflation gap. If* $\sigma_\varphi^2 < \dfrac{ab\tilde{\theta}_B}{c\left(1 + b^2\tilde{\theta}_B\right)}$, *an increase in Ω_G enlarges the inflation gap.*

iii) *For any m_0, uncertain government preferences aggravate the gap between expected output and its target level:* $\dfrac{\partial|y^e - y^*|}{\partial\Omega_G} > 0$.

Proof. Substitute $\sigma_\rho^2 = 0$ into Eqs. (15-19). □

Part i) of Proposition 2.4 states that there is either inflation bias or negative inflation bias in the equilibrium. The logic is similar to the model with certain preferences. Monetary multiplicative uncertainty causes the attenuation monetary effect, equal to $\frac{\hat{W}_m}{\hat{W}}$. Similar to the case of certain preferences, to change the average fiscal instrument by $\frac{\hat{W}_m}{\hat{W}}\frac{m_0}{c}$ would be enough to compensate the influence of monetary attenuation effect on inflation. Nevertheless, the government with any preferences has a competing target of output. As the government does not want to change considerably the output level, there is the under-reaction to the monetary attenuation effect. The average size of this under-reaction is given by the term $\frac{-\Lambda_B a\left(a+bc\right)\tilde{\theta}_G - ac^2\Omega_G\Lambda_B\left(-\left(a+bc\right)\right)}{\hat{W}}\frac{m_0}{c}$ in equation (15). This under-reaction gives rise to the gap between expected inflation and its target, while the equilibrium average change in fiscal instrument gives rise to the gap between expected output and the target output. The signs of the inflation and output gaps depend on the sign of m_0. If m_0 is positive, negative inflation bias with low inflation and high output arises. It means that uncertain government preferences to some extent eliminate the inflation bias problem, which is caused by uncertainty about monetary multiplicative uncertainty. If m_0 is negative, uncertainty leads to a standard inflation bias.

Parts ii) and iii) of Proposition 2.4 characterize the effects of preference uncertainty on the absolute values of the output and inflation gaps. To better understand these findings, let us firstly note that the size of monetary attenuation effect, $\frac{\hat{W}_m}{\hat{W}}$, depends positively on Ω_G. This means that an increase in preference uncertainty aggravates the attenuation effect of monetary policy. The explanation is as follows. As we have seen in Section 2.3, if $m_0 > 0$ and preferences are certain, the equilibrium fiscal instrument is decreasing and convex function of government type. This means that under uncertain preferences the average fiscal policy is looser than the policy of the government with the average preferences. Thus, the central bank decreases m in accordance with its reaction function. This signifies an aggravation of the attenuation effect in comparison with the certain preferences model. If $m_0 < 0$, the fiscal instrument under certain preferences is an increasing concave function of the government preferences. Thus, the average fiscal policy is tighter than the policy chosen by the government with the average preferences. The central bank reacts to this by an increase in m. As the attenuation effect in this case implies the rise of m, we can conclude that uncertainty about preferences again aggravates the attenuation effect.

The gap between expected output and the target output is defined by the government reaction to this attenuation effect. The change in the fiscal instrument is proportional to the size of the attenuation effect. From here we can conclude, that the absolute value of the output gap is also proportional to the attenuation effect. Thus, an increase in preference uncertainty always aggravates the output gap which is caused by monetary multiplicative uncertainty.

The gap between expected inflation and its target is defined by the average fiscal under-reaction to the monetary attenuation effect. The under-reaction of the government with preferences θ_G is proportional to $\Lambda_B\left(\tilde{\theta}_G - c^2\omega(\theta_G)\right)a(a+bc)$. As there is no fiscal multiplicative uncertainty, the following equation holds:

$$\tilde{\theta}_G - c^2\omega(\theta_G) = \theta_G \frac{c^2 + (a+bc)^2\tilde{\theta}_G}{c^2 + (a+bc)^2\theta_G} \tag{20}$$

From (20) we can see that the coefficient $\tilde{\theta}_G - c^2\omega(\theta_G)$ is non-negative and depends positively on θ_G. This means that stronger the government preferences for output the less reaction to the monetary attenuation effect. Moreover, function $\tilde{\theta}_G - c^2\omega(\theta_G)$ is concave in θ_G. The average under-reaction of the government to the monetary attenuation effect, $\left.\frac{\Lambda_B\left(\tilde{\theta}_G - c^2\Omega_G\right)a(a+bc)}{\hat{W}}\right|_{\sigma_\rho^2=0}$, defines the gap between expected inflation and inflation target. The size of this gap depends on the variance of the government preferences, Ω_G. The sign of this relation is defined by the extent of monetary uncertainty. If the monetary multiplicative uncertainty is strong and $\sigma_\varphi^2 > \frac{ab\tilde{\theta}_B}{c\left(1+b^2\tilde{\theta}_B\right)}$, a decrease in Ω_G leads to an increase in the under-reaction. This means that more uncertain preferences lower the gap between expected inflation and the inflation target. On the contrary, if monetary uncertainty is weak and $\sigma_\varphi^2 < \frac{ab\tilde{\theta}_B}{c\left(1+b^2\tilde{\theta}_B\right)}$, an increase in uncertainty about the government preferences leads to an increase in the gap between the expected and target inflation rates.

4.4 Uncertain policy effects and uncertain fiscal preferences

After discussion of the polar cases in the previous subsections, we now proceed to the general framework. The characteristics of the equilibrium with uncertain preferences and uncertain policy effects are summarized in the following Proposition:

Proposition 5. *For given (σ_ρ^2, σ_φ^2, Ω_G), there exist $\lambda_2^* \geq \lambda_3^* \geq \lambda_1^*$, such that:*

i) $\pi^e \geq \pi^*$ *if and only if* $\frac{m_0}{\tau_0} \leq \lambda_1^*$;

ii) $y^e \geq y^*$ *if and only if* $\frac{m_0}{\tau_0} \geq \lambda_2^*$;

iii) $\frac{\partial(y^e - y^*)}{\partial\Omega_G} \geq 0$ *if and only if* $\frac{m_0}{\tau_0} \geq \lambda_3^*$, *and*

$$\frac{\partial\left(\pi^e-\pi^*\right)}{\partial\Omega_G}\geq 0 \text{ if and only if } \left(\frac{m_0}{\tau_0}-\lambda_3^*\right)\left(\sigma_\varphi^2-\frac{ab\tilde{\theta}_B}{c\left(1+b^2\tilde{\theta}_B\right)}\right)>0;$$

$$\text{where } \lambda_1^*=\frac{c^2\left(\Lambda_G+a\sigma_\rho^2\Omega_G\left(a+2bc\right)\right)\left(ab\tilde{\theta}_B-c\Lambda_B\right)}{\Lambda_B\left(c^2\Lambda_G+\tilde{\theta}_G a\left(a+bc\right)+ac^2\Omega_G\left(bc\left(\sigma_\rho^2-1\right)-a\right)\right)},$$

$$\lambda_2^*=\frac{c\left(1+\Lambda_B\right)}{\Lambda_B}\frac{\left(\Lambda_G+a\sigma_\rho^2\Omega_G\left(a+2bc\right)\right)}{\left(1+\Omega_G\left(\left(a+bc\right)^2+b^2c^2\sigma_\rho^2\right)\right)}\geq 0,$$

$$\lambda_3^*=\frac{c\sigma_\rho^2\left(a^2+abc\left(1+b^2\tilde{\theta}_B\right)+\Lambda_B\left(a\left(a+bc\right)-b^2c^2\right)\right)}{\Lambda_B\left(a\left(a+bc\right)+\sigma_\rho^2b^2c^2\right)}.$$

Proof. See Eqs. (15-19). □

Parts i) and ii) of Proposition 2.5 state that if both policy effects are uncertain, there are three possible economic situations: inflation bias, deflation bias or negative inflation bias. If $\frac{m_0}{\tau_0}\leq\lambda_1^*$, there is the inflation bias problem in the economy: the expected rate of inflation exceeds its target level ($\pi^e\geq\pi^*$), while the expected rate of output is below its target level ($y^e\leq y^*$). If $\lambda_1^*<\frac{m_0}{\tau_0}\leq\lambda_2^*$, there is the deflation bias problem in the economy: the expected rate of inflation and output are below their target levels ($\pi^e\leq\pi^*, y^e\leq y^*$). If $\frac{m_0}{\tau_0}>\lambda_2^*$, the expected rate of output exceeds its target level ($y^e\geq y^*$), while the expected level of inflation is below its target level ($\pi^e\leq\pi^*$), which means that there is the negative inflation bias problem in the economy. Similar to the model with certain preferences, the deflation bias is possible only if both multiplicative shocks are present and $\frac{m_0}{\tau_0}\in(\lambda_1^*,\lambda_2^*)$.

Uncertainty about the government preferences influences the thresholds λ_1^* and λ_2^*. It is easy to show that an increase in uncertainty about the government preferences lowers λ_2^*. The effect of preference uncertainty on the value of λ_1^* depends on the sign of its value. If λ_1^* is positive, an increase in Ω_G leads to a further increase in λ_1^*. If λ_1^* is negative, an increase in Ω_G leads to a further decrease in λ_1^*.

Part iii) of Proposition 2.5 defines the effect of preference uncertainty on the equilibrium output and inflation. The effect of preference uncertainty on expected output is positive if $\frac{m_0}{\tau_0}>\lambda_3^*$ and negative if $\frac{m_0}{\tau_0}<\lambda_3^*$. This means that if $\frac{m_0}{\tau_0}<\lambda_1^*$ and the equilibrium is characterized by inflation bias with negative output gap, an increase in preference uncertainty leads to a further increase in the absolute value of this gap. If $\frac{m_0}{\tau_0}>\lambda_2^*$ and the equilibrium is characterized by the negative inflation bias with positive output gap, an increase in preference uncertainty also leads to a further increase in the absolute value of this gap. If $\frac{m_0}{\tau_0}\in(\lambda_1^*,\lambda_2^*)$, there might be non-monotonous effect of preference uncertainty on the output gap. Thus, there may be a positive effect of preference uncertainty.

The effect of preference uncertainty on expected inflation depends not only on the value of $\frac{m_0}{\tau_0}$, but also on the extent of monetary multiplicative uncertainty. For example, if $\frac{m_0}{\tau_0} > \lambda_3^*$, the equilibrium is characterized by negative gap between expected inflation and its target. The effect of Ω_G depends on the value of σ_ϕ^2. If $\sigma_\varphi^2 > \frac{ab\tilde{\theta}_B}{c(1+b^2\tilde{\theta}_B)}$, an increase in Ω_G leads to an increase in expected inflation and consequently, to a decrease in the absolute value of the inflation gap. Similarly, if $\sigma_\varphi^2 < \frac{ab\tilde{\theta}_B}{c(1+b^2\tilde{\theta}_B)}$, an increase in Ω_G leads to a decrease in expected inflation and consequently, to an increase in the absolute value of the inflation gap.

5 Welfare analysis

In previous Section we have analyzed the effects of uncertainty on inflation and output gaps. Now we are going to discuss the optimal design of policy decision-making under uncertainty. For this purpose, we have to define the welfare criterion. Following the consensus in the literature, we assume that this criterion is represented by the following social loss function:

$$L_S = E\left[(\pi - \pi^*)^2 + \theta_W (y - y^*)^2\right], \tag{21}$$

where θ_W characterizes the social preferences for output in comparison to inflation. Using equations (1) and (2) together with their expectations, we rewrite the social loss function in the following way:

$$\begin{aligned} L_S = {} & (\pi^e - \pi^*)^2 + \theta_W (y^e - y^*)^2 + \left(1 + b^2\theta_W\right)\left(\sigma_\varphi^2 m^2 + c^2\sigma_\rho^2\overline{\tau}^2\right) + \\ & + \left[c^2\left(1+\sigma_\rho^2\right) + \theta_W\left(b^2c^2\sigma_\rho^2 + (a+bc)^2\right)\right] E\left(\tau - \overline{\tau}\right)^2 \end{aligned} \tag{22}$$

As we can see, the first term in social loss represents the squared expected gap between the equilibrium inflation and its target level. The second term is the squared gap between the equilibrium output and its target level. The previous sections show that these gaps originate from sub-optimal reaction of policymakers to multiplicative uncertainty. We have also discussed the effect of preference uncertainty on these gaps. The third term in (22), equal to $(1 + b^2\theta_W)\left(\sigma_\varphi^2 m^2 + c^2\sigma_\rho^2\overline{\tau}^2\right)$, represents the weighted volatility of inflation and output, created by multiplicative shocks. The last term represents the expected loss from uncertainty about fiscal preferences and is proportional to the variance of fiscal instrument $E\left(\tau - \overline{\tau}\right)^2$.

According to (16), the gap between the action of the government with preferences θ_G and the average government action is proportional to $(\omega(\theta_G) - \Omega_G)$. Thus, the variance of government actions is proportional to the variance of variable $\omega(\theta_G)$. As this function is non-linear, we cannot derive its variance explicitly without specifying the distribution of preferences. Because of that, we restrict our attention to economies with sufficiently weak uncertainty about government preferences, meaning that θ_G is fairly close to its mean $\tilde{\theta}_G$. This assumption allows us to linearize $\omega(\theta_G)$ around $\tilde{\theta}_G$ and to use a simple expression for its variance without specifying the exact distribution functions:

Assumption 6. *Let θ_G be fairly close to the mean $\tilde{\theta}_G$, so we can use the following approximations:*

i) $\omega(\theta_G) \approx \omega\left(\tilde{\theta}_G\right) + \omega'\left(\tilde{\theta}_G\right)\left(\theta_G - \tilde{\theta}_G\right) + \frac{1}{2}\omega''\left(\tilde{\theta}_G\right)\left(\theta_G - \tilde{\theta}_G\right)^2$

ii) $\Omega_G = E(\omega(\theta_G)) \approx \omega\left(\tilde{\theta}_G\right) + \frac{1}{2}\omega''\left(\tilde{\theta}_G\right)\sigma_\theta^2$

iii) $E(\omega(\theta_G) - \Omega_G)^2 \approx \left(\omega'\left(\tilde{\theta}_G\right)\right)^2 \sigma_\theta^2,$

where σ_θ^2 is the variance of government preferences.

Assumption 6 allows us to get the social loss function explicitly. Using this assumption, we substitute equilibrium policies (15-17) and equilibrium gaps from (18-19) into equation (22). This gives us the expression of social loss which depends on preference parameters θ_W, θ_B, $\tilde{\theta}_G$, the variances of multiplicative shocks σ_φ^2, σ_ρ^2 and the government preference uncertainty, measured by σ_θ^2. Minimization of this loss with respect to θ_B, $\tilde{\theta}_G$ would give the optimal policymakers preferences or an optimal policy design, defined as follows:

Definition 7. The optimal policy design is a vector of policymakers preferences $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right) \equiv \left(\theta_B^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right), \tilde{\theta}_G^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)\right)$ such that:

$$\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right) = \arg\min_{\Theta} \tilde{L}_S\left(\theta_B, \tilde{\theta}_G, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right),$$

where $\Theta = \left(\theta_B, \tilde{\theta}_G\right) > 0$ and $\tilde{L}_S\left(\theta_B, \tilde{\theta}_G, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ is the expected social loss in equilibrium.

Social planner which cannot influence the extent of multiplicative uncertainty, uncertainty about the government preferences or the form of policy interaction, assigns the central bank with preferences $\theta_B^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ and chooses the average type of government $\tilde{\theta}_G^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$. Unfortunately, it is impossible to find the closed-form solution of the optimal program in the general model. Thus, we use the following procedure. Firstly, we find the optimal policy preferences in the model with the only multiplicative shock (either fiscal or monetary). After that we investigate the effects of sufficiently small increase in uncertainty about the other multiplicative shock and about the government preferences on the optimal values of θ_B and $\tilde{\theta}_G$. The situation without multiplicative uncertainty is trivial. As we have seen in the previous section, in this situation the governments with any preferences choose the same value of fiscal instrument. As a result, there is no fiscal policy uncertainty and no gaps between the equilibrium values of inflation and output and their targets. Thus, for any policy preferences social loss is equal to zero. Multiplicative uncertainty of any type creates the gaps between the equilibrium levels of output and inflation and their targets, volatility of output and inflation and uncertainty about fiscal policy. This justifies the need to assign the proper policymakers which could minimize the losses created by uncertainty. Following the logic of previous sections, we start with fiscal multiplicative uncertainty (Proposition 8) and proceed with monetary multiplicative uncertainty (Proposition 9).

Proposition 8. *Let* $\sigma_\rho^2 > 0$*. Then the optimal preference parameters* θ_B^* *and* $\tilde{\theta}_G^*$ *are such that:*

i) $\theta_B^*\big|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} = 0$ *and* $\tilde{\theta}_G^*\Big|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} = \dfrac{a\theta_W}{a+bc\left(1+b^2\theta_W\right)}$;

ii) $\dfrac{\partial\theta_B^*}{\partial\sigma_\theta^2}\bigg|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$ *and* $\dfrac{\partial\tilde{\theta}_G^*}{\partial\sigma_\theta^2}\bigg|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$;

iii) $\dfrac{\partial\theta_B^*}{\partial\sigma_\varphi^2}\bigg|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$ *and* $\dfrac{\partial\tilde{\theta}_G^*}{\partial\sigma_\varphi^2}\bigg|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$*, if and only if* $\dfrac{m_0}{c\tau_0} < \dfrac{\sigma_\rho^2 c^2\left(1+b^2\theta_W\right)}{a^2\theta_W+\sigma_\rho^2 c^2\left(1+b^2\theta_W\right)}$.

Proof. See Appendix B. □

Part i) of Proposition 8 defines the optimal policy design without monetary multiplicative uncertainty and without preference uncertainty. As this situation is equivalent to Di Bartolomeo, Giuli and Manzo (2009), the optimal policy preferences are the same as in their model. The optimal choice of policymakers implies that both of them should be more conservative than the society. This is explained by the time-inconsistency problem. Both reaction functions (5) and (6) show that the policymaker have the incentive to push output up by inflation surprise. To avoid this, they should be sufficiently conservative. Moreover, the central bank should be more conservative than the government ($\tilde{\theta}_G^* > \theta_B^*$) and should not worry about output ($\theta_B^* = 0$). There are two reasons for this. The first reason is that the central bank cannot influence output in equilibrium. The second reason is the overreaction of the central bank to the attenuation in fiscal policy. As we have discussed earlier, fiscal multiplicative uncertainty leads to a fiscal attenuation effect which is expressed by a drop in fiscal instrument. The central bank faces the time-inconsistency problem and overreacts to this drop by too loose monetary policy. The overreaction of the central bank is proportional to its preference for output θ_B. Thus, assigning an absolutely conservative central bank without preference for output ($\theta_B^* = 0$) allows to avoid this overreaction. As a result, the expected inflation is kept at its target level.

Part ii) of Proposition 8 states that an increase in preference uncertainty makes the optimal conservativeness of both the central bank and the government lower. Earlier we have seen that preference uncertainty not only creates the uncertainty about fiscal policy, but also deteriorates the gaps caused by the fiscal multiplicative shock. This effect was summarized by variable Ω_G in Section 4. From Part ii) of Assumption 6, it immediately follows that Ω_G depends positively on preference uncertainty σ_θ^2 and negatively on the average government preferences $\tilde{\theta}_G$. Thus, in order to smooth the negative effect of σ_θ^2 on the output and inflation gaps, an increase in $\tilde{\theta}_G$ is needed. Moreover, from Part iii) of Assumption 6 along with the properties of function $\omega(\theta_G)$, we can conclude that the variances of $\omega(\theta_G)$ and $\tau(\theta_G)$ depend positively on σ_θ^2 and negatively on $\tilde{\theta}_G$. This again makes it socially desirable to assign the less conservative government. Nevertheless, higher average government preferences and more active fiscal policy lead to higher volatility of both inflation and output because of fiscal multiplicative shocks. This, however, can be compensated by a less conservative central bank. As a result, both $\tilde{\theta}_G^*$ and θ_B^* increase with an increase in σ_θ^2.

Part iii) of Proposition 8 explores the effect of a small increase in monetary multiplicative uncertainty on the optimal preferences. As we can see, this effect depends on the relation between policy action under certainty or, in other words, on the relation between inflation and output targets. If the inflation target is small relative to the output target, such that m_0 is small relative to τ_0, an increase in σ_φ^2 leads to a decrease in the optimal conservativeness for both policymakers. To explain this, we need to study the effect of monetary multiplicative uncertainty on the output and inflation gaps and the equilibrium policy actions. It is easy to show from (9- 12) that if $\frac{m_0}{c\tau_0} < \frac{\sigma_\rho^2 c^2 (1+b^2\theta_W)}{a^2\theta_W + \sigma_\rho^2 c^2 (1+b^2\theta_W)}$, a small increase in σ_φ^2 leads to an increase in the equilibrium monetary policy action and consequently, to an increase in the inflation gap. The fiscal policy becomes less active, output drops, the absolute value of the output gap increases. An increase in θ_G and θ_B would help to restore the output close to the target level without a large increase in inflation, as far as m_0 is sufficiently small. The opposite happens if m_0 is large and $\frac{m_0}{c\tau_0} > \frac{\sigma_\rho^2 c^2 (1+b^2\theta_W)}{a^2\theta_W + \sigma_\rho^2 c^2 (1+b^2\theta_W)}$. In this case an increase in σ_φ^2 leads to a decrease in the equilibrium monetary action and to an increase in the equilibrium fiscal policy action. As a result, the expected inflation decreases, while the expected output increases. As the initial equilibrium was characterized by inflation bias, an increase in σ_φ^2 lowers the absolute values of both gaps. Thus, more conservative government and the central bank can be assigned in order to lower τ and m and to decrease the volatility created by the corresponding multiplicative shocks.

The properties of the optimal policy design in economy with monetary multiplicative uncertainty are summarized by the following proposition:

Proposition 9. *Let* $\sigma_\varphi^2 > 0$

i) $\theta_B^*\big|_{\sigma_\theta^2=0,\sigma_\rho^2=0} = \frac{ab\theta_W^2}{c+b\theta_W(a+bc)}$ *and* $\tilde{\theta}_G^*\Big|_{\sigma_\theta^2=0,\sigma_\rho^2=0} = \frac{a\theta_W}{a+bc}$;

ii) $\left.\frac{\partial\theta_B^*}{\partial\sigma_\theta^2}\right|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$ *and* $\left.\frac{\partial\tilde{\theta}_G^*}{\partial\sigma_\theta^2}\right|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$ *;*

iii) $\left.\frac{\partial\theta_B^*}{\partial\sigma_\rho^2}\right|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$ *and* $\left.\frac{\partial\tilde{\theta}_G^*}{\partial\sigma_\rho^2}\right|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$*, if and only if* $m_0 > 0$.

Part i) of Proposition 9 describes the optimal policymakers preferences for the situation when only monetary multiplicative uncertainty is present. Similar to the situation with fiscal multiplicative uncertainty, the central bank should be more conservative than the government, and both should be more conservative than society. The reason is again the time inconsistency problem and the impossibility for the central bank to influence output in equilibrium. Contrary to the previous situation with fiscal multiplicative uncertainty, the central bank should not be absolutely conservative and have to worry about output ($\theta_B^* > 0$). The logic here is as follows. According to reaction function (6), the monetary multiplicative uncertainty forces the central bank to decrease its actions proportionally (monetary attenuation effect). This means that its incentives to stimulate output also weaken and time inconsistency problem becomes less pronounced. As a result, there is no need to assign the fully conservative central bank. Moreover, as $\frac{a\theta_W}{a+bc} > \frac{a\theta_W}{a+bc\left(1+b^2\theta_W\right)}$, the government under monetary multiplicative uncertainty should be also less conservative than under fiscal multiplicative uncertainty. To better understand this finding, let us remind that the reaction of the government to the attenuation effect in monetary policy depends negatively on its preferences for output θ_G. As this reaction creates the output gap, the society would be better off if the government with higher θ_G is assigned.

Part ii) of Proposition 9 states that the effects of preference uncertainty under monetary multiplicative uncertainty are the same as under fiscal multiplicative uncertainty. An increase in preference uncertainty lowers the optimal conservativeness of both the central bank and the government, making $\tilde{\theta}_G^*$ and θ_B^* higher. The intuition is similar. An increase in σ_θ^2 leads to an increase in Ω_G, in output gap and in the volatility of fiscal policy actions. An increase in the average government preference for output is needed to compensate for these discrepancies. An increase in θ_B^* is needed to lower the volatility of output and inflation, created by monetary multiplicative uncertainty.

Part iii) of Proposition 9 explores the effect of a small increase in fiscal multiplicative uncertainty on the optimal preferences. As we can see, this effect depends on sign of m_0, which, in turn, depends on the relation between the inflation and output targets. If the inflation target is sufficiently high and monetary policy under certainty is relatively loose ($m_0 > 0$), an increase in σ_ρ^2 leads to an decrease in the optimal conservativeness of both policymakers. The intuition is straightforward. If m_0 is positive, the effect of monetary multiplicative uncertainty is a decrease in m and an increase in τ, resulting in too high output and too low inflation. If fiscal multiplicative uncertainty arises in such a situation, fiscal policy becomes less expansionary. As a result, the expected inflation drops further. To avoid this drop in inflation, less conservative government and central bank should be assigned. If inflation target is sufficiently low and monetary policy under certainty is relatively tight ($m_0 < 0$), an increase in σ_ρ^2 leads to an increase in the optimal conservativeness of both policymakers. If m_0 is negative, the equilibrium with monetary multiplicative uncertainty is characterized by looser monetary policy and tighter fiscal policy, which lead to inflation bias. If we add fiscal multiplicative uncertainty, the government becomes less active, which helps to keep output closer to its target but pushes inflation up. To avoid this increase in inflation, more conservative authorities are needed and both $\tilde{\theta}_G^*$ and θ_B^* decrease.

6 Conclusion

This paper contributes to the existing literature on monetary and fiscal policy under uncertainty. In particular, we study the role of uncertain government preferences for policy interaction.

We show, that if the fiscal and monetary policy effects are certain, uncertainty about government preferences does not affect the equilibrium. In case of fiscal multiplicative uncertainty, uncertainty about the government preferences lowers output, increases inflation and thereby aggravates the inflation bias problem. Monetary multiplicative uncertainty can create either the inflation bias problem or negative inflation bias problem. Uncertain government preferences aggravate the problem by enlarging the absolute value of the output gap, while the effect on the inflation gap depends on the extent of uncertainty about the monetary policy effectiveness and may be beneficial. If both the policy effects are uncertain, the impact of uncertain government preference depends not only on the extent of multiplicative uncertainty, but also on the inflation and output targets. As a result, preference uncertainty may lower the absolute values of output and inflation gaps, created by multiplicative uncertainty.

Our welfare analysis is restricted to the small extents of preference uncertainty which allows us to derive the welfare function explicitly without specifying the exact distribution function. Nevertheless, higher extents of uncertainty can be also studied, probably with the use of numerical methods. Another restriction of our study is that we deal only with uncertainty about the policy effects on inflation. The direct effects of fiscal policy on output are treated as known. Nevertheless, it seems that in reality the knowledge about these policy effects is also far from completeness. Thus, incorporating uncertainty about the effects on output is a promising avenue for future research. Moreover, the problem of different forms of strategic interaction is beyond the scope of our paper: we consider that the government and the central bank conduct their policies simultaneously and independently. The analysis of the influence of uncertain government preferences on macroeconomic policy under various forms of strategic interaction is left for future studies.

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A Proof of Propositions 8 and 9

Proposition 8 i) and 9i)

The vector of optimal weights $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ solves the following system of first order conditions:

$$D_\Theta \tilde{L}_S\left(\Theta, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right) = \underline{0}, \tag{23}$$

where D is the derivative operator. Substituting zeros in stead of corresponding σ_j^2, $j \in \{\varphi, \rho, \theta\}$, we get the system which can be solved for $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$. Normally, there are several pairs of roots but only the roots listed in i) Parts of Propositions 8 and 9 assure that the Hessian matrix of $\tilde{L}_S$ is positive semi-definite and that the found solution $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ minimizes the social loss.

Proposition 8 ii-iii) and 9ii-iii)

To find the signs of corresponding derivatives, we use

$$\frac{\partial \theta_k^*}{\partial \sigma_j^2} = -\frac{|H_{kj}|}{|H|}, \tag{24}$$

where $k \in \{B, G\}$, $|H|$ is the determinant of the Hessian matrix and $|H_{kj}|$is the determinant of the Hessian matrix where the k-th column was replaced by the $D^2_{\Theta,\sigma_j^2}\tilde{L}_S\left(\Theta, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$, computed for $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$. As $|H|$ is non-negative, the sign of $\dfrac{\partial \theta_k^*}{\partial \sigma_j^2}$ corresponds to the sign of $(-1)\,|H_{kj}|$. Calculations are available upon request.