The value of Information in Segmented Economies

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Abstract

We study the social value of information in a general two-region model, which captures three important characteristics of international markets: segmentation of fundamentals, globalization of markets and informational asymmetry between regions. We show that the welfare properties of information in segmented economy differ significantly from their counterpart in a one-region model. We prove that Angeletos and Pavan (2007)'s result (the negative gap between efficient and equilibrium degree of coordination is sufficient for welfare to increase in precision of private information economies with strategic substitutability), does not hold in segmented economy. We also prove that a high (low) gap between efficient and equilibrium distributions does not imply a positive (negative) value of information, in a two-region distorted economy, when externalities are strong enough. Moreover, we obtain the conditions under which the regional value of information differs from its social value. These findings indicate when the regional information policy could be inefficient. After discussing the general model, we illustrate our findings with a number of examples.

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Key words: *strategic complementarity, strategic substitutability, public information, private information, value of information, segmented economy*

1 Introduction

The social value of information has been broadly discussed in the literature. Starting from the seminal paper by Morris and Shin (2002), most researchers which deal with this issue consider economic environments with common fundamental shocks (e.g. Angeletos and Pavan (2007), Cornand and Heinemann (2008), Ui and Yoshizawa (2015), Roca (2010), James and Lawler (2012), Walsh (2013), etc.). Some authors assume that the common shocks are complemented with agent-specific idiosyncratic shocks (e.g. Hellwig and Venkateswaran (2009), Venkateswaran (2014), Bergemann, Heumann and Morris (2015), Amador and Weill (2010)). Irrespective of the precise economic environment, all these studies investigate the role of information in closed economies, for which such shock structure may be reasonable. Nevertheless, as far as the focus is shifted to international context, these assumptions do not seem reliable any more.

In global economy, shocks are neither entirely common nor agent-specific; more likely, they are segmented or, in other words, country-specific. The segmentation or *regionalization*[1] of shocks across the international economy has been documented by a vast literature on international business cycles (e.g.

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[1]term by Heathcote and Perri (2004)

Heathcote and Perri (2002), Heathcote and Perri (2004)), capital flows (Tille and van Wincoop (2014), Tille and Van Wincoop (2010)), international asset trade (Bhamra, Coeurdacier and Guibaud (2014), Devereux and Sutherland (2011)). For example, the segmentation of fundamentals can come from uncorrelated shocks to non-asset incomes across countries, country-specific productivity innovations (Tille and van Wincoop (2014)) or country-specific transaction costs (Bhamra, Coeurdacier and Guibaud (2014)).

The segmentation of shocks across the world has not found a lot of attention in the literature on the welfare properties of information. To the best of our knowledge, the only exception is the study of Arato and Nakamura (2013), who extend the beauty-contest model of Morris and Shin (2002) to a two-region version with uncorrelated country-specific fundamentals. Nevertheless, Arato and Nakamura (2013) assume that the beauty contest is not global, but region-specific, meaning that private agents have incentive to mimic the average actions only in their home region, not in the whole economy. In fact, Arato and Nakamura (2013) model two autarky economies, for which the only link is informational spillover, as the signals about region-specific fundamentals are dispersed world-wide. Thus, the model does not capture the full degree of globalization in international trade and investments, which is documented by many researchers.

Apart from segmentation of fundamental shocks, many authors confirm that there exists the informational asymmetry between countries. There is a huge literature which shows that locals have an informational advantage over foreigners (Bae, Stulz and Tan (2008), Ferreira et al. (2017), Dvořák (2005), Van Nieuwerburgh and Veldkamp (2009)). Another strand of literature shows that the informational asymmetry between countries may explain some empirical findings in international portfolio allocation (see Thapa, Paudyal and Neupane (2013) for the survey). The theoretical literature on the social value of information also discusses a specific kind of informational asymmetry. For example, Cornand and Heinemann (2008) and James and Lawler (2012) assume that the public signal reaches only a rate of population. This type of asymmetry is different from informational asymmetry in international finance literature, as all the agents have the same probability of access to this information, while in most financial studies agents have higher probability to get their home information.

The goal of this research is to fill the gap in the literature and to define the value of information in international economies. For this purpose, we explore a stylized two-country model, which captures three main characteristics of international markets: segmentation of fundamentals, informational asymmetry between countries and global strategic complementarity or substitutability in private actions. Basically, this general model is a two-country extension of the model of Angeletos and Pavan (2007), where the whole population is split between two countries with country-specific fundamentals. Informational asymmetry between countries is modeled by the different composition of private signals. We assume that private signals contain information only about the home fundamental shocks. The only source of information about the foreign shock is a public signal, which is available to all the agents in the economy. Thus, each private agent receives three signals: one public signal about the home fundamental shock, one public signal about the foreign fundamental shock and one private signal about the home fundamental shock.

For this general model, we derive social and regional loss functions, and show that social and regional welfare depends not only on the average gaps between equilibrium and optimal actions and their volatility, as in Angeletos and Pavan (2007), but also on relative gaps between regions. Our contribution is two-fold. First of all, we test the findings of Angeletos and Pavan (2007), who derive the complete classification of homogeneous economies according to their welfare properties of information. We show that the crucial parameter, which affects the value of information in two-regional economy, is the externality created by the cross-sectional dispersion. If this externality is absent, almost all the findings of Angeletos and Pavan (2007) stay relevant for segmented economy. Nevertheless, we find that some of results from Angeletos and Pavan (2007) do not hold in segmented economy in case of strategic substitutability. For example, we show that a negative gap between efficient and equilibrium degrees of coordination is not sufficient for private information to be socially needed. The reason is the fact that in this economy equilibrium coordination is inefficient not only inside the region, but also between regions. If strategic substitutability is relatively high, an increase in the precision of private information may force agents to coordinate more inside the region, but this will have a negative effect on inter-regional coordination. If the externality created by the cross-sectional dispersion is sufficiently high, all the findings of Angeletos and Pavan (2007) about the social value of information may be violated, because this externality implies the higher weight of inter-regional gaps in social loss function. For example, the negative externality of the inter-regional gap in private action implies that the social value of private information may be negative. The presence of private information, which is available only to the inhabitants of one region, automatically creates the inter-regional asymmetry in private actions. If the society values this asymmetry negatively, an increase in the precision of this information may lower social welfare, even if it would be valuable in homogeneous societies. Similarly, the positive externality of the cross-sectional dispersion may make the social value of public information negative, even if the efficient extent of coordination is positive.

The second contribution of the paper is that we characterize the regional and inter-regional value of information. When doing so, we detect the situations, in which the social value of information differs from its regional value. These differences in information structures which are optimal from the social and regional point of view, would help to detect the risks of inefficient information policy, if it is conducted by the local authorities. For example, in economies with globally efficient strategic complementarity and positive externality of cross-sector dispersion, the regional value of public information may be negative, while its social value is positive. This happens because the regional value of inter-regional gap is higher than its social value. Thus, if the social authority is the sender of public information about his home region, he or she would publish too little information.

Finally, we illustrate our findings with a number of examples which are widely used in the literature on social value of information.

The rest of the paper is organized as follows. The general two-country framework is introduced in the next Section. Sections 3-5 deal with the equilibrium allocation, social optimum and regional optimum, correspondingly. In Section 6 we discuss the welfare properties of information in several examples. The last section concludes.

2 Framework

In order to study the value of information in segmented economies, we extend the model of Angeletos and Pavan (2007) into a two-region version. We assume that the unit mass of private agents forms the population of an economy. This population is divided into two groups, each of which inhabits one region. Let $i \in [0,1]$ denote the index of a private agent. Agents with index $i \in [0,n] \equiv G_1$ belong to group 1 (or live in region 1) and agents with index $i \in (n,1] \equiv G_2$ belong to group 2 (or live in region 2). Thus, the size of region 1 is equal to n, while the size of region 2 is equal to $(1-n)$.

Let k_i^j denote the action taken by agent i who lives in region j. Then the average private action in this region, K_j, is given by the following expression:

$$K_j \equiv \frac{1}{n_j} \int_{i \in G_j} k_i^j \, \mathrm{d}i$$

The average private action in the economy, $K \equiv \int_{j \in \{1,2\}} \int_{i \in G_j} k_i^j \, \mathrm{d}i \, \mathrm{d}j$, is equal to the weighted average private actions in both regions:

$$K \equiv nK_1 + (1-n)\,K_2$$

The dispersion of private actions in the economy $\sigma_k^2 \equiv \int_{j\in\{1,2\}} \int_{i\in G_j} \left(k_i^j - K\right)^2 \mathrm{d}i\,\mathrm{d}j$ is defined by the dispersion of private actions in both regions and by the gap in private actions between the regions:

$$\sigma_k^2 = n\sigma_1^2 + (1-n)\,\sigma_2^2 + n\,(1-n)\,(K_1 - K_2)^2\,, \tag{1}$$

where $\sigma_j \equiv \left({}^{1}\!/_{n_j} \int_{i\in G_j} \left(k_i^j - K_j\right)^2 \mathrm{d}i\right)^{1/2}$ is the standard deviation of private actions in region j, $j \in \{1,2\}$.

The payoff of private agent i living in region j depends on his action k_i^j, average private action K, the standard deviation of private actions in the economy σ_k , fundamental parameter θ^j and is written by the following function:

$$u_i^j = U\left(k_i^j, K, \sigma_k, \theta^j\right), \tag{2}$$

Fundamental θ^j can be interpreted as a technological parameter. This variable is normally distributed with mean $\mu^j = 0$ and variance $\sigma_{\theta,j}^2$. For simplicity, we assume that fundamentals in different region are uncorrelated. Thus, there are only local idiosyncratic technological shocks, without technological spillovers between regions. Nevertheless, the payoff function (2) allows for the global strategic effects, as the private payoffs depend on the global average action K.

Following the methodology of Angeletos and Pavan (2007), we assume that payoff function $U\left(k_i^j, K, \sigma_k, \theta^j\right)$ is a quadratic function with $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = U_\sigma\left(k_i^j, K, 0, \theta^j\right) = 0$. This means that payoff function is separable in dispersion term and the other variables. In other words, the dispersion has only non-strategic effect on private payoffs. As we will see later, this implies that the equilibrium private actions do not depend on the dispersion. Thus, the payoff function can be rewritten in the following form:

$$u_i^j = U\left(k_i^j, K, 0, \theta^j\right) + \tfrac{U_{\sigma\sigma}}{2}\sigma_k^2, \tag{3}$$

Moreover, we assume that the payoff function is concave in private actions ($U_{kk} < 0$). Moreover, $U_{kK} < -U_{kk}$, where U_{kK} measures the strategic effect in private actions. If $U_{kK} = 0$, the private actions are independent of the average actions in the economy. If $U_{kK} > 0$, the private payoff is higher when the private action is closer to the average action K. Thus, there is strategic complementarity in private actions and private agents have the incentive to do what others do. If $U_{kK} < 0$, the private payoff is higher when the distance between a private action and the average action in the economy is larger. As the private payoff depends on the average for the whole economy, there is a global strategic effect. The alternative version would be a local strategic effect, if the private payoff was linked to the average actions in the home region. The additional assumption is $U_{kk} + 2U_{kK} + U_{\sigma\sigma} < 0$.

The effect of dispersion in private actions can have any sign. If $U_{\sigma\sigma} > 0$, there is a positive private value of dispersion in private actions. This, for example, is the characteristic of a beauty-contest model described by Morris and Shin (2002). If $U_{\sigma\sigma} < 0$, there is a negative private value of dispersion in private actions, as in Walsh (2013). If $U_{\sigma\sigma} = 0$, private payoffs do not depend on the dispersion. Despite of the sign of this variable, we assume that $U_{kk} + U_{\sigma\sigma} < 0$. The model of Angeletos and Pavan (2007) is a special case of ours and can be obtained by choosing $n = 1$.

We assume that $U_k\,(0,0,0,0) = U_K\,(0,0,0,0) = 0$. This assumption simplifies considerably the derivations, but does not affect the conclusions about the value of public and private information. Moreover, without lack of generality, $U_{k\theta} > 0$. This assumption means that private agents have an incentive to keep their actions close to their home fundamentals.

We assume that private agents do not know the true values of the fundamentals θ^1 and θ^2. Instead of the perfect information, private agents have an excess to several imperfect signals about the fundamentals. All agents in the economy observe two public signals about the fundamentals:

$$y^j = \theta^j + \eta^j, j \in \{1, 2\} \tag{4}$$

where $\eta^j \sim N(0, \sigma^2_{y,j})$ is the noise of public signal y^j with variance $\sigma^2_{y,j}$. Thus, $\sigma^{-2}_{y,j}$ is the precision of a public signal about the fundamental shock in region j. If $\sigma^{-2}_{y,j} = 0$, the prediction value of this information is zero. This is equivalent to the absence of such public information. Two public signals are uncorrelated, meaning that covariance of two noises is equal to zero ($\text{Cov}\left(\eta^1, \eta^2\right) = 0$).

Thus, the public information about fundamental θ^j consists of public signal y^j and the prior information about the fundamental μ^j. In what follows, we use the composite signal z^j to denote all public information about the fundamental shock in region j:

$$z^j \equiv \frac{\sigma^{-2}_{y,j} y^j + \sigma^{-2}_{\theta,j} \mu^j}{\sigma^{-2}_{y,j} + \sigma^{-2}_{\theta,j}}.$$

Dispersion of the noise in this composite signal is equal to $\sigma^2_{z,j} = \left(\sigma^{-2}_{y,j} + \sigma^{-2}_{\theta,j}\right)^{-1}$ and precision of public information is equal to $\sigma^{-2}_{z,j} = \left(\sigma^{-2}_{y,j} + \sigma^{-2}_{\theta,j}\right)$. This composite signal is observed by all agents in the economy; there is no difference in the access to public information between the agents in different regions. The only difference in information available to private agents concerns their private information. Private agent i living in region j observes private signal x^j_i about the true value of θ^j:

$$x^j_i = \theta^j + \varepsilon^j_i, \tag{5}$$

where $\varepsilon^j_i \sim i.i.d.N(0, \sigma^2_{x,j})$ is the noise of this private signal and $\sigma^2_{x,j}$ stands for its variance. Thus, value $\sigma^{-2}_{x,j}$ depicts the precision of private information in region j. We suppose that agents in region j do not observe any private signal about the foreign fundamental shock θ^{-j}.

Private agents use their private signals and two composite public signals to form their expectations about the fundamentals:

$$E\left[\left(\begin{array}{c} \theta^j \\ \theta^{-j} \end{array}\right) \middle| x^j_i, z^j, z^{-j}\right] = \left(\begin{array}{c} \delta^j z^j + \left(1 - \delta^j\right) x^j_i \\ z^{-j} \end{array}\right), \tag{6}$$

where $\delta^j = \dfrac{\sigma^{-2}_{z,j}}{\sigma^{-2}_{z,j} + \sigma^{-2}_{x,j}}$. According to (6), a private agent from region j uses his own private signal x^j_i and public signal z^j to derive his expectations about θ^j. As the agent has no private information about fundamentals in the other region, his expectations about θ^{-j} are equal to public information about θ^{-j}. These expectations are used by private agents to choose their actions. The equilibrium private actions are defined in the next section.

3 Equilibrium

Private agents simultaneously choose their actions, which maximize their payoff (2). Before proceeding to the equilibrium under imperfect information, we start with the properties of the equilibrium in an economy where all the agents know the true values of fundamentals.

3.1 Equilibrium with complete information

The equilibrium with complete information is characterized by a pair of strategies $\left(\kappa^1, \kappa^2\right)$: $\mathbb{R}^2 \to \mathbb{R}^2$such that

$$\kappa^j\left(\Theta^j\right) = \arg\max_{k^j} U\left(k^j, \tilde{K}\left(\theta^j, \theta^{-j}\right), \tilde{\sigma}_k\left(\theta^j, \theta^{-j}\right), \theta^j\right), \tag{7}$$

where $\Theta^j = \left(\theta^j, \theta^{-j}\right)$ is a vector of fundamental shocks, $\tilde{K}\left(\theta^j, \theta^{-j}\right) = \int_{j\in\{1,2\}} \int_{i\in G_j} \kappa^j\left(\theta^j, \theta^{-j}\right) \mathrm{d}i\, \mathrm{d}j$ is the average private actions under complete information and $\tilde{\sigma}_k\left(\theta^j, \theta^{-j}\right) \equiv \left(\int_{j\in\{1,2\}} \int_{i\in G_j} \left(\kappa^j\left(\theta^j, \theta^{-j}\right) - K\left(\theta^j, \theta^{-j}\right)\right)^2 \mathrm{d}i\, \mathrm{d}j\right)^{1/2}$ is the standard deviation of private actions in equilibrium.

Similarly to one-region model of Angeletos and Pavan (2007), the equilibrium private strategies under complete information are linear over the fundamentals and are given by the following expression:

$$\kappa_j\left(\Theta\right) = \kappa_{j,j}\theta^j + \kappa_{j,-j}\theta^{-j}, \tag{8}$$

where $\kappa_{j,j}$ is the equilibrium weight of the home fundamental factor and $\kappa_{j,-j}$ is the equilibrium weight of the foreign fundamental factor in private actions in region j. These equilibrium weights are as follows:

$$\kappa_{j,j} = \kappa - \alpha\kappa(1 - n_j) \tag{9}$$

$$\kappa_{j,-j} = \alpha\kappa(1 - n_j), \tag{10}$$

where $\kappa = \dfrac{-U_{k\theta}}{U_{kk} + U_{kK}}$ and $\alpha = \dfrac{U_{kK}}{-U_{kk}}$.

By assumptions made before, $U_{k\theta}$ is positive while expression $U_{kk} + U_{kK}$ is negative. This implies that κ is positive. The sign of α coincides with the sign of U_{kK} and is positive, if there is strategic complementarity, and negative, if there is strategic substitutability. As $-U_{kK} > U_{kk}$, the value of α belongs to the interval $(-\infty, 1)$. This means that the weight of home fundamental shock in private actions in region j is positive. This reflects the incentive of private agents to keep their actions close to their home fundamentals. Despite the foreign fundamentals do not have the direct effect on the private payoffs, the agents also react to the foreign fundamental shock, as far as there is the strategic effect and $\alpha \neq 0$. If $\alpha > 0$ and private actions are characterized by strategic complementarity, the agents in region j also have the incentive to keep their actions close to the private actions in foreign region $-j$. As private agents in region $-j$ align their actions to foreign fundamentals θ^{-j}, strategic complementarity forces agents in region j to put a positive weight $\kappa_{j,-j}$ to this fundamental shock. The stronger strategic complementarity and the larger foreign region, the higher weight of foreign fundamentals is attached by private agents in region j. This leads to an equivalent decrease in the weight of home fundamental in private actions. If there is strategic substitutability and $\alpha < 0$, the agents want to differentiate their actions with the actions of others. This imply a negative weight of foreign fundamentals in private actions and an increase in the weight of home fundamentals.

The redistribution of the whole weight κ between the two fundamentals depends on the extent of strategic effect α and on the region size n_j. If we take the limiting case with $n_j = 1$, we get the one-region model of Angeletos and Pavan (2007) with the equilibrium private actions:

$$\kappa_j\left(\theta^j, \theta^{-j}\right) = \kappa\theta^j \tag{11}$$

If there is strategic complementarity ($\alpha > 0$), the weight of the local fundamentals in private actions is lower in two-regional model. The agents redistribute this weight toward the foreign shock, as there is strategic complementarity between regions. If there is strategic substitutability ($\alpha < 0$), the weight of the local fundamentals in private actions is higher in two-regional model. The agents want to keep their actions far from the foreign actions, as there is strategic substitutability between regions. Thus, the agents attach a negative weight to the foreign fundamentals and increase the weight of local fundamentals.

The average private actions in two-region economy, $\bar{\kappa}$, are proportional to the average value of fundamentals in both regions:

$$\bar{\kappa} \equiv n\kappa_1\left(\theta^1,\theta^2\right) + (1-n)\,\kappa_2\left(\theta^1,\theta^2\right) = \kappa\left(n\theta^1 + (1-n)\,\theta^2\right)$$

Nevertheless, this model in general version should not be treated as an average model with actions $\bar{\kappa}$ and fundamentals $\bar{\theta} \equiv \left(n\theta^1 + (1-n)\,\theta^2\right)$, because there is asymmetry between regions, which may affect private payoffs. This asymmetry can be illustrated by the gap between private actions in the regions:

$$\kappa_1\left(\theta^1,\theta^2\right) - \kappa_2\left(\theta^1,\theta^2\right) = \kappa\,(1-\alpha)\left(\theta^1 - \theta^2\right)$$

This gap vanishes only if the fundamental parameters are equal in two regions. As far as $\theta^1 \neq \theta^2$, private actions differ in two regions. Even if there is no dispersion in private actions inside the regions, the gap between average actions creates the dispersion between regions and the dispersion of private actions in the whole economy, according to equation (1). If private agents do not care about the dispersion and $U_{\sigma\sigma} = 0$, this does not affect the private payoffs. If there is the negative private value of dispersion and $U_{\sigma\sigma} < 0$, the gap between the regions creates the negative effect on private payoffs. If there is a positive private value of dispersion and $U_{\sigma\sigma} > 0$, the gap between the regions creates the positive effect on private payoffs. By assumption, this is a second-order effect which does not influence the equilibrium private actions. Nevertheless, this effect has a crucial impact on the social and regional welfare and is a crucial determinant of the social and local value of private and public information, as it will be shown later.

3.2 Equilibrium with incomplete information

Under incomplete information, private agents do not know the true value of fundamental shocks. Thus, they choose their actions in order to maximize their expected payoff given their information set. The information set for any agent consists of three elements. The first element is the private signal about the home fundamental. The second and the third elements are the public signal about their home fundamentals and the public signal about the foreign fundamentals. Formally, equilibrium with incomplete information is a pair of strategies (k_1, k_2): $\mathbb{R}^3 \to \mathbb{R}^2$ such that

$$k^j\left(x^j, z^j, z^{-j}\right) = \arg\max_{k'} E\left[U\left(k', K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \theta^j\right)\middle|\, x^j, z^j, z^{-j}\right], \tag{12}$$

where $\Theta = \left(\theta^1,\theta^2\right)$ is a vector of fundamentals, $Z = \left(z^1, z^2\right)$ is a vector of public information, $K\left(\Theta, Z\right) = \int_{j\in\{1,2\}}\int_{x^j} k^j\left(x^j, z^j, z^{-j}\right) \mathrm{d}P\left(x^j\middle|\,\theta^j, z^j\right)\mathrm{d}j$ is the average private action in equilibrium and $\sigma_k\left(\Theta, Z\right) = \left(\int_{j\in\{1,2\}}\int_{x^j}\left(k^j\left(x^j, z^j, z^{-j}\right) - K\left(\theta^j, \theta^{-j}, z^j, z^{-j}\right)\right)^2 \mathrm{d}P\left(x^j\middle|\,\theta^j, z^j\right)\mathrm{d}j\right)^{1/2}$ is the equilibrium standard deviation of private actions.

The first-order condition, which describes the equilibrium strategies (12), is as follows:

$$k^j\left(x^j, z^j, z^{-j}\right) = E\left[\kappa_j\left(\Theta\right) + \alpha_{j,j}\left(K_j\left(\Theta, Z\right) - \kappa_j\left(\Theta\right)\right) + \alpha_{j,-j}\left(K_{-j}\left(\Theta, Z\right) - \kappa_j\left(\Theta\right)\right)\middle|\, x^j, z^j, z^{-j}\right]. \tag{13}$$

where $K_j\left(\Theta, Z\right)$is the average private action in region j for given fundamental shocks Θ and public information Z, $\alpha_{j,j} = \alpha - \alpha(1-n_j)$ and $\alpha_{j,-j} = \alpha(1-n_j)$.

According to (12), the optimal action of a private agent depends on his expectations about the optimal action under complete information $\kappa_j(\Theta)$, the expected gap between the average actions under incomplete and complete information in his home region, $(K_j(\Theta, Z) - \kappa_j(\Theta))$, and the expected gap between the average actions under incomplete and complete information in the foreign region, $(K_{-j}(\Theta, Z) - \kappa_j(\Theta))$. Value $\alpha_{j,j}$ measures the impact of the home gap in private actions on the decision of the agent. In other words, $\alpha_{j,j}$ is the regional extent of coordination. Similarly, the value $\alpha_{j,-j}$ measures the impact of the foreign gap in private actions on the decision of any agent in region j. Thus, $\alpha_{j,-j}$ is the inter-regional extent of coordination. If there is strategic complementarity ($\alpha > 0$), both regional and inter-regional extents of coordination are positive and agents are willing to mimic the average actions in both regions. The larger region, the stronger desire to mimic its average actions both inside and between regions. If there is strategic substitutability, both regional and inter-regional extents of coordination are negative.

The first-order condition (12) gives the linear equilibrium strategy of private agents. This strategy is described in the following proposition.

Proposition 1. *In a linear equilibrium, the strategy of private agents is as follows:*

$$k^j\left(x^j, z^j, z^{-j}\right) = \kappa_{j,j}\left(\gamma^j z^j + \left(1-\gamma^j\right)x^j\right) + \kappa_{j,-j}z^{-j}, \tag{14}$$

where γ^j is the relative weight of regional public information given by:

$$\begin{aligned} \gamma^j &= \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha_{j,j}}{1-\left(1-\delta^j\right)\alpha_{j,j}} + \frac{\left(1-\delta^j\right)\alpha_{j,-j}}{1-\left(1-\delta^j\right)\alpha_{j,j}}\frac{\kappa_{-j,j}}{\kappa_{j,j}} = \\ &= \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha_{j,j}}{1-\left(1-\delta^j\right)\alpha_{j,j}} + \frac{\left(1-\delta^j\right)\alpha_{j,-j}}{1-\left(1-\delta^j\right)\alpha_{j,j}}\frac{\alpha_{-j,j}}{1-\alpha_{j,-j}} \end{aligned} \tag{15}$$

Proposition 1 shows that private agents in two-region economy use the information about both regions, as far as both $\kappa_{j,j}$ and $\kappa_{j,-j}$ are non-zero. We have shown earlier that the weight of home information $\kappa_{j,j}$ is positive, while the weight of foreign information $\kappa_{j,-j}$ is positive only in case of strategic complementarity. It is negative, if there is strategic substitutability and zero, if there is no strategic effect. As the only source of information about the foreign fundamental shock is public signal z^{-j}, the weight of this signal in private action in region j coincides with the weight of foreign fundamental in private actions under complete information. As there are two sources of information about the home fundamentals, the private agent redistributes the entire weight of home information $\kappa_{j,j}$ between them. Parameter $\gamma^j \in [0,1]$ shows the relative weight of home public information, while $\left(1-\gamma^j\right)$ measures the relative weight of a private signal in the entire use of home information.

It can be easily seen that the relative weight of public home information is equal to the relative precision of public information δ^j if and only if the strategic effect is absent and $\alpha = 0$. If there is strategic complementarity, the relative weight of public signal in actions exceeds its relative precision. This can be explained by the desire of private agents to mimic the actions of others. The use of a public signal allows them to better predict the actions of others and the use of public signal increases even if this does not allow the agents to keep their actions closer to the relevant home fundamentals. If there is strategic substitutability, the agents have the desire to differentiate their actions from the actions of others. Thus, they decrease the weight of the home public signal in their actions to a level which is lower than the relative precision of public information. An increase in relative precision of public information and strategic complementarity leads to an increase in the relative weight of public home information in private actions.

The effect of region size n_j on the relative weight of public information is non-linear. The following

Corollary summarizes the effect of regional size on the relative weight of its home public information:

Corollary 1. *The effect of region size on the relative weight of its public information is as follows:*

1. *In case of strategic complementarity,* $\frac{\partial \gamma^j}{\partial n^j} > 0$ *if and only if* $n^j < \min\left(\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right);1\right)$*;*

2. *In case of strategic substitutability,* $\frac{\partial \gamma^j}{\partial n^j} < 0$ *if and only if* $n^j > \max\left(\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right);0\right)$*.*

The first part of Corollary 3.2 describes the properties of γ^j in case of strategic complementarity. It can be easily shown that threshold $\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right)$ is larger than 1, if precision of local public information in region j is relatively high and $\delta^j > \frac{\alpha}{1+\alpha}$. In such situation, the relative weight is increasing in region size and the relative weight of local public information in two-region economy is lower than in one-region economy. In this case, public information is a very good predictor of home fundamentals; thus, its weight in home private actions is initially very high. When there are two regions instead of one, strategic complementarity forces the agents to switch from their home public information to foreign public information. As a result, they redistribute the use of public information as a instrument of coordination towards the foreign signal. If precision of local public information is low and $\delta^j < \frac{\alpha}{1+\alpha}$, threshold $\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right)$ is lower than 1 and the relative weight of public information is a hump-shaped function of n_j. Thus, the relative weight of public local information may be higher in two-region economy than in a one-region economy. In this case, the weight of home public information is not that high in a one-region model due to the relatively low precision of this information. Strategic complementarity between regions makes the inhabitants of the foreign region willing to react to the public information about region j. The population of region j knows this and may want to mimic the actions of foreigners by increasing the weight of the home public information, despite its relatively bad quality. Thus, in two-region economy agents in a large region may attach higher weight to their home public signals than they would in a one-region world. Worth to note, that this effect is present only if precision of public information is relatively low and the region is relatively large. To illustrate the reasoning, we provide the equilibrium strategies in one-region economy, which can be obtained from ours by taking $n = 1$. In this model, the equilibrium action of private agents is the function of their private signal and the public signal:

$$k^j\left(x^j, z^j\right) = \kappa\left(\hat{\gamma}^j z^j + \left(1-\hat{\gamma}^j\right)x^j\right), \tag{16}$$

where $\hat{\gamma}^j$ is the relative weight of regional public information in one-region economy and is given by:

$$\hat{\gamma}^j = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha}{1-\left(1-\delta^j\right)\alpha} \tag{17}$$

The second part of Corollary 3.2 summarizes the properties of γ^j in case of strategic substitutability. If precision of local public information is high and $\delta^j > \frac{-\alpha}{1-\alpha}$, threshold $\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right)$ is negative. Thus, an increase in region size leads to a decrease in the relative weight of local public information. Consequently, the relative weight of local public information in two-region economy is higher than in one-region economy. The reasoning is straightforward. If the quality of home public information is relatively good, the agents would use it to keep their actions close to their home fundamentals. The inter-regional strategic substitutability means that the part of the whole population is not going to use this information. Thus, the agents may increase their use of the home public information without suffering from increased coordination. If the quality of public information is relatively bad and $\delta^j < \frac{-\alpha}{1-\alpha}$, the agents may prefer to rely more on their private information to keep their actions close to the fundamentals. In this case, the relative gains of using the home public information are small and the weight of public information is a hump-shaped function of n. Thus, the relative weight of public local information may be lower in a two-region economy than in a one-region economy.

Taking into account the equilibrium strategy under incomplete information (14) and under complete information (8), we can show that the average actions in region j under incomplete information are equal to the sum of average actions in this region under complete information and the weighted errors of the public signals $\left(z^j - \theta^j\right)$ and $\left(z^{-j} - \theta^{-j}\right)$:

$$K_j\left(\Theta^j, Z^j\right) = \kappa_j\left(\Theta\right) + \kappa_{j,j}\gamma^j\left(z^j - \theta^j\right) + \kappa_{j,-j}\left(z^{-j} - \theta^{-j}\right) \tag{18}$$

This gives the average actions in the whole economy:

$$K\left(\Theta, Z\right) = \bar{\kappa}\left(\Theta\right) + \sum_{j\in\{1,2\}}\left(n_j\kappa_{j,j}\gamma^j + \left(1 - n_j\right)\kappa_{-j,j}\right)\left(z^j - \theta^j\right)\,\mathrm{d}j, \tag{19}$$

where $\left(n_j\kappa_{j,j}\gamma^j + \left(1 - n_j\right)\kappa_{-j,j}\right)$ is the average weight of signal z^j in private actions.

The gap between the average actions in the two regions is equal to the sum of the gap between the regions under complete information and the relative errors of the public signals:

$$K_1\left(\Theta, Z\right) - K_2\left(\Theta, Z\right) = \kappa_1\left(\Theta\right) - \kappa_2\left(\Theta\right) + \left(\kappa_{1,1}\gamma^1 - \kappa_{2,1}\right)\left(z^1 - \theta^1\right) - \left(\kappa_{2,2}\gamma^2 - \kappa_{1,2}\right)\left(z^2 - \theta^2\right), \tag{20}$$

where $\left(\kappa_{j,j}\gamma^j - \kappa_{-j,j}\right)$ is the relative weight of signal z^j in actions in region j in comparison to its weight in region $-j$. Thus, the errors in public signals create the deviation of the average actions from their values under complete information. The use of imperfect private signals creates the dispersion of the actions inside the regions. The dispersion of private actions in region j is equal to

$$\sigma_j^2 = \kappa_{j,j}^2\left(1 - \gamma^j\right)^2\sigma_{x,j}^2 \tag{21}$$

The welfare properties of the noise in public and private information are discussed in the next section.

4 Social welfare analysis

In this section, we discuss the social value of public and private information in segmented economies. We start with the description of socially efficient allocations under complete and incomplete information. After that we derive the social loss function and find the impact of precision of public and private information on this welfare criterion.

The social welfare is the sum of all private payoffs in the economy:

$$W \equiv \int_{j\in\{1,2\}}\int_{i\in S^j} U\left(k^j\left(x^j, z^j, z^{-j}\right), K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \theta^j\right)\,\mathrm{d}i\,\mathrm{d}j$$

This welfare can be rewritten as a sum of two components:

$$W = \omega\left(K_1, K_2, \theta^1, \theta^2\right) + \frac{W_{\sigma\sigma}}{2}\left(n_1\sigma_1^2 + n_2\sigma_2^2\right), \tag{22}$$

where $\omega\left(K_1, K_2, \theta^1, \theta^2\right)$ is the component, which depends on the regional average private actions, the average private actions in the whole economy and fundamental shocks. Term $\frac{W_{\sigma\sigma}}{2}\left(n_1\sigma_1^2 + n_2\sigma_2^2\right)$ is the component which depends on the dispersion of private actions inside the regions. Coefficient $W_{\sigma\sigma} = U_{\sigma\sigma} + U_{kk}$ measures the social value of dispersion inside the regions. As it is negative, the dispersion in private actions lowers the social welfare and is undesirable from the social perspective. Worth to note, that the social value of dispersion inside regions is negative irrespective of the private value of dispersion. The component which depends on the averages is given by the following expression:

$$\omega\left(K_1, K_2, \theta^1, \theta^2\right) = n_1 U\left(K_1, K, 0, \theta^1\right) + n_2 U\left(K_2, K, 0, \theta^2\right) + \frac{U_{\sigma\sigma}}{2} n_1 n_2 \left(K_1 - K_2\right)^2 \tag{23}$$

The first term on the right-hand part in (23) is the payoff of agents in the first region, if all of them choose action K_1. The second term is the payoff of the agents in the second region, if they choose action K_2. The last term shows the global gains of private agents due to the gap in actions between the regions. If $U_{\sigma\sigma} < 0$ and there is a negative private value of dispersion, the social welfare is negatively related to the gap between regions. In other words, society values negatively the difference between regions. If $U_{\sigma\sigma} > 0$ and there is a positive private value of dispersion, society values positively the difference between regions. Thus, the social value of the gap between regions coincides with the private value of dispersion.

4.1 Social optimum under complete information

To find the social optimum, we assume that the social planner decides on the private actions for given values of fundamental shocks $\left(\theta^1, \theta^2\right)$. As the society gets a negative value of dispersion inside the regions, the social planner chooses the same action for all agents which live in the same region. Thus, the efficient allocation with complete information is a pair of strategies (κ_1^*, κ_2^*): $\mathbb{R}^2 \to \mathbb{R}^2$ such that

$$\kappa_j^*\left(\theta_j, \theta_{-j}\right) = \arg\max_{K_j} \omega\left(K_j, \kappa_{-j}^*, \theta^j, \theta^{-j}\right), \tag{24}$$

where $\omega\left(K_j, \kappa_{-j}^*, \theta^j, \theta^{-j}\right)$ is welfare component (23). The socially efficient actions for agents in region j are linear over two fundamental shocks:

$$\kappa_j^*\left(\theta^j, \theta^{-j}\right) = \kappa_{j,j}^* \theta^j + \kappa_{j,-j}^* \theta^{-j}, \tag{25}$$

where the weights of fundamentals are

$$\kappa_{j,j}^* = \kappa^* - \left(1 - n_j\right) \hat{\kappa} \tag{26}$$

$$\kappa_{j,-j}^* = \left(1 - n_j\right) \hat{\kappa}, \tag{27}$$

where $\kappa^* = \frac{(U_{k\theta} + U_{K\theta})}{-W_{KK}}$, $\hat{\kappa} \equiv -\frac{(U_{k\theta} + U_{K\theta})}{W_{KK}} + \frac{U_{k\theta}}{W_{\sigma\sigma}}$ and $W_{KK} = U_{kk} + 2U_{kK} + U_{KK} < 0$.

Thus, the socially optimal private actions under complete information are the weighted sum of the two fundamentals. The optimal distribution in a one-region model can be obtained from (25–27) by choosing $n = 1$:

$$\kappa_j^*\left(\theta^j, \theta^{-j}\right) = \kappa^* \theta^j.$$

Thus, the relation between the weight of the home fundamental in a one-region model and its weight in a two-region model (26) is defined by the value of $\hat{\kappa}$. If $\hat{\kappa} > 0$, the optimal weight of local fundamentals is lower in two-region social optimum in comparison to one-country model. The weight of foreign fundamentals is positive. This happens if the social aversion to variance in private actions is stronger than the desire to reach the fundamentals, such that $W_{\sigma\sigma} < \frac{U_{k\theta}}{(U_{k\theta} + U_{K\theta})} W_{KK}$. This condition is equivalent to $1 - \frac{W_{KK}}{W_{\sigma\sigma}} > -\frac{U_{K\theta}}{U_{k\theta}}$. As we will see later, value $\alpha^* = 1 - \frac{W_{KK}}{W_{\sigma\sigma}}$ characterizes the socially optimal degree of coordination. If the optimal degree of coordination is high, the social planner is ready to sacrifice the closeness of private actions to the local fundamentals in order to vanish the difference between regions. As a result, the weight of fundamentals is redistributed from the local shock to the foreign one. The extent of this redistribution depends positively on the size of foreign region. Thus, the efficient distribution in a two-region model is shifted to the fundamentals in the largest region.

If $\hat{\kappa} < 0$, the optimal weight of local fundamentals is higher in two-region social optimum in comparison to one-country model. The weight of foreign fundamentals is negative. This happens if the social aversion to variance in private actions is not very high, such that $W_{\sigma\sigma} > \frac{U_{k\theta}}{(U_{k\theta}+U_{K\theta})} W_{KK}$. This is equivalent to relatively small efficient degree of coordination, $\alpha^* < -\frac{U_{K\theta}}{U_{k\theta}}$. In this case, the social planner does not care much about the variance in private actions. Thus, the planner is ready to stretch the distance between regions in order to diminish the gap between the local fundamentals and the local private actions. In this case, the private actions in two regions are shifted apart from each other and there is a substantial gap between them.

The gap between efficient actions in two regions in a model with complete information is proportional to the gap between fundamental shocks:

$$\kappa_1^* \left(\theta^1, \theta^2\right) - \kappa_2^* \left(\theta^1, \theta^2\right) \quad = \quad \left(\kappa^* - \hat{\kappa}\right) \left(\theta_1 - \theta_2\right),$$

where coefficient $(\kappa^* - \hat{\kappa}) = -\frac{U_{k\theta}}{W_{\sigma\sigma}}$ is positive. If there is a huge social aversion to dispersion and the absolute value of $W_{\sigma\sigma}$ is high, the value of $(\kappa^* - \hat{\kappa})$ and the gap between the regions vanish. If the social aversion to dispersion is modest, the value of $(\kappa^* - \hat{\kappa})$ and the gap between regions are large.

The average efficient action in the economy is proportional to the average value of fundamental shock:

$$\bar{\kappa}^* \equiv n\kappa_1^* + (1-n)\,\kappa_2^* = \kappa^* \left(n\theta^1 + (1-n)\,\theta^2\right).$$

This value does not depend on the private or social value of dispersion. Nevertheless, the average efficient actions in a model with incomplete information do depend on these parameters, as we will see in the next subsection.

4.2 Social optimum with incomplete information.

An efficient allocation with incomplete information is a pair of strategies (k_1^*, k_2^*): $\mathbb{R}^3 \to \mathbb{R}^2$ such that

$$\left\{k_1^* \left(x^1, Z\right), k_2^* \left(x^2, Z\right)\right\} = \arg \max_{k'(x,Z)} E\left[W\left(k\left(x, Z\right), K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \Theta\right)\right], \tag{28}$$

where $k\left(x, Z\right) = \left\{k_1\left(x^1, Z\right), k_2\left(x^2, Z\right)\right\}$ is a feasible set of private actions, $K\left(\Theta, Z\right) = \int_{j\in\{1,2\}} \int_{x^j} k_j\left(x^j, Z\right) \, \mathrm{d}P\left(x^j \middle| \Theta, Z\right) \mathrm{d}j$ and $\sigma_k\left(\Theta, Z\right) = \left(\int_{j\in\{1,2\}} \int_{x^j} \left(k_j\left(x^j, Z\right) - K\left(\Theta, Z\right)\right)^2 \mathrm{d}P\left(x^j \middle| \Theta, Z\right) \mathrm{d}j\right)^{1/2}$.

This implies the following first-order condition:

$$k_j^* \left(x^j, z^j, z^{-j}\right) = E\left[\kappa_j^*\left(\Theta\right) + \alpha_{j,j}^* \left(K^j\left(\Theta, Z\right) - \kappa_j^*\left(\Theta\right)\right) + \alpha_{j,-j}\left(K^{-j}\left(\Theta, Z\right) - \kappa_{-j}^*\left(\Theta\right)\right) \middle| x^j, z^j, z^{-j}\right], \tag{29}$$

The first-order condition shows that the efficient strategy for any agent in region j is the sum of his expected efficient action under complete information $\kappa_j^*\left(\Theta\right)$ and the expected gaps between the average actions and the corresponding efficient average actions under complete information for both regions. Value $\alpha_{j,j}^* \equiv \alpha^* - (1-n_j)\dfrac{W_{KK} - U_{kk}}{-W_{\sigma\sigma}}$ is the efficient extent of coordination inside the region and $\alpha_{j,-j}^* \equiv \dfrac{W_{KK} - U_{kk}}{-W_{\sigma\sigma}}(1-n_j)$

is the efficient inter-regional extent of coordination, $\alpha^* = 1 - \frac{W_{KK}}{W_{\sigma\sigma}}$ is the efficient extent of coordination in a one-region model.

The inter-regional efficient extent of coordination is positive if the marginal social utility of average actions decreases slower than the marginal private utility of private actions, meaning that $W_{KK} > U_{kk}$. This happens if the private value of coordination is sufficiently high, such that $U_{kK} > -\frac{U_{KK}}{2}$. In this case, the social value of coordination between regions is high and the efficient inter-regional coordination is positive, $\alpha^*_{j,-j} > 0$. The regional degree of coordination diminishes by the value of the inter-regional degree of coordination and is lower than in a one-region economy. This redistribution of coordination between regions is higher for the larger size of the other region. Thus, the efficient allocation implies that the actions are shifted to the average actions in a larger region. If the private value of coordination is low, $U_{kK} < -\frac{U_{KK}}{2}$, the marginal social utility of average actions decreases faster than the marginal private utility of private actions, meaning that $W_{KK} < U_{kk}$. In this case, the efficient extent of inter-regional coordination is negative and the efficient extent of coordination inside the region is higher than in a one-region model.

The first-order condition (29) gives the linear efficient strategy of private agents. This strategy is described by the following proposition.

Proposition 2. *The linear efficient strategy of private agents is as follows:*

$$k^*_j\left(x^j, z^j, z^{-j}\right) = \kappa^*_{j,j}\left(\gamma^*_j z^j + \left(1-\gamma^*_j\right)x^j\right) + \kappa^*_{j,-j} z^{-j}, \tag{30}$$

where γ^*_j is the efficient relative weight of regional public information given by:

$$\gamma^*_j = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha^*_{j,j}}{1-\left(1-\delta^j\right)\alpha^*_{j,j}} + \frac{\left(1-\delta^j\right)\alpha^*_{j,-j}}{1-\left(1-\delta^j\right)\alpha^*_{j,j}}\frac{\kappa^*_{-j,j}}{\kappa^*_{j,j}} \tag{31}$$

In a one-region model the efficient action are as follows:

$$k^*_j\left(x^j, z^j\right) = \kappa^*\left(\gamma^*_j z^j + \left(1-\gamma^*_j\right)x^j\right), \tag{32}$$

with

$$\hat{\gamma}^*_j = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha^*}{1-\left(1-\delta^j\right)\alpha^*} \tag{33}$$

Comparison of these strategies with the equilibrium in a one-region model shows that the equilibrium is socially efficient if $\kappa = \kappa^*$ and $\alpha = \alpha^*$. In this case, the equilibrium and efficient distribution under complete information are the same and the efficient degree of coordination coincides with the equilibrium degree of coordination. In a two-region economy, these conditions are necessary but not sufficient for equilibrium to be optimal. Condition $\kappa = \kappa^*$ assures that the average actions in the equilibrium and in the optimum coincide under complete information. Nevertheless, this does not guarantee that the distribution of these averages between regions is efficient. Condition $\alpha = \alpha^*$ assures that the average degrees of coordination are efficient, but it is not sufficient for both regional and inter-regional degrees of coordination to be efficient. Comparison of equilibrium strategies (14) with socially efficient strategies (30) gives the following sufficient condition for the efficiency of equilibrium allocation:

Proposition 3. *Equilibrium in a two-regional model is socially efficient if and only if $\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} = 0$.*

Thus, equilibrium strategies in a two-region model is efficient if they are socially efficient in a one-region model and the private value of dispersion and the social value of the gap between two regions are equal to zero. This finding demonstrates higher importance of parameter $U_{\sigma\sigma}$ in a two-region model in comparison with a one-region model. In order to better understand this finding, we consider three possible sources of inefficiency in segmented economy: the gap between equilibrium and efficient degrees of coordination $\alpha^* - \alpha$, the gap between efficient and equilibrium average allocation under complete information $\kappa^* - \kappa$ and the externality of dispersion in private actions $U_{\sigma\sigma}$. The positive gap between α^* and α means that equilibrium coordination degrees in the model are insufficiently low, both inside and between regions. If $\kappa^* > \kappa$, the agents respond insufficiently to the shocks in both home and foreign fundamentals. Thus, the first two sources of inefficiency equally strike the agents reaction to home and foreign variables. On the contrary, the externality caused by dispersion in private actions creates an additional asymmetry. It can be easily shown that the negative externality ($U_{\sigma\sigma} < 0$) makes the regional degree of coordination inefficiently low and inter-regional degree of coordination insufficiently high. Moreover, this leads to the positive gap between the efficient and equilibrium weights of home shocks under complete information and to a negative gap between the efficient and equilibrium weights of foreign shocks in private actions. As we will see in the next section, the social welfare in a segmented economy depends on the gap in actions between two regions. Thus, the asymmetry created by this externality, can considerably change the welfare properties of information in a two-region model. In the next subsection we derive the social loss function, which is then used to study the welfare properties of public and private information.

4.3 Social loss function

The expected value of social welfare (22) can be written as:

$$EW^S = \omega^S\left(\kappa_1^*, \kappa_2^*, \theta^1, \theta^2\right) - \mathscr{L}_S^*,$$

where $\omega^S\left(\kappa_1^*, \kappa_2^*, \theta^1, \theta^2\right)$ is the first-best social welfare and $\mathscr{L}_S^*$ is the social loss which arises due to the gap between the equilibrium and social optimum. The value of this loss is as follows:

$$\begin{aligned} \mathscr{L}_S^* = & \frac{|W_{KK}|}{2} Var\left(K - \bar{K}^*\right) + \frac{n(1-n)}{2}|W_{\sigma\sigma}| Var\left(K_1 - \kappa_1^* - (K_2 - \kappa_2^*)\right) \\ & + \frac{|W_{\sigma\sigma}|}{2}\left(n\sigma_1^2 + (1-n)\sigma_2^2\right) \end{aligned} \tag{34}$$

Thus, equation (34) reveals the sources of inefficiency in the described economy. The first source of inefficiency is the gap between the equilibrium average actions and the socially efficient average actions. The variance of this gap is denoted by $Var\left(K - \bar{K}^*\right)$ in equation (34). Coefficient $\frac{|W_{KK}|}{2}$ measures the impact of this variance on the social loss. The second term in social loss comes from the possible asymmetry between the regions. The value $K_1 - \kappa_1^* - (K_2 - \kappa_2^*)$ measures the relative gap between the average regional actions and the corresponding optimal actions. If the gaps between average and optimal actions are different for the regions, the asymmetry arises and social welfare deviates further from the first-best. Coefficient $\frac{n(1-n)}{2}|W_{\sigma\sigma}|$ measures the importance of the inter-regional asymmetry for social planner. Finally, the social loss comes from the variance in private actions in both regions, which is measured by σ_1^2 and σ_2^2. For the larger region, concerns about its private actions dispersion are stronger.

The gap between the equilibrium and first-best allocations can arise because of two reasons. The first reason is the inefficient structure of the economy, such that equilibrium under complete information is not efficient. The second reason is incomplete information. These two reasons can be partially separated from each other. For example, the gap between average equilibrium and efficient actions can be represented as the sum of the gap between the average equilibrium actions under complete and incomplete information and the gap between the equilibrium actions under complete information and average efficient actions:

$$K-\bar{K}^{*}=(K-\bar{\kappa})+\left(\bar{\kappa}-\bar{K}^{*}\right)$$

The variance of this sum is equal to the sum of variances of two gaps and their doubled covariance. As value $\left(\bar{\kappa}-\bar{K}^{*}\right)$ is the gap between the equilibrium and efficient action under complete information, its value does not depend on the information available to the agents. Thus, the first component in (34) can be represented by the sum of two terms, one of which is independent of the information quality, while the other is defined by the precision of public and private information available to agents. The same is true for the second component in social loss function. The dispersion of private actions arises only under incomplete information and thus, it is fully defined by the information precision. As a result, we can rewrite the social loss (34) as the sum of component L_S^0, which is independent of precision of public and private information, and component L_S, which depends on the precisions:

$$\mathscr{L}_S^{*}=L_S^0+L_S,$$

where component L_S is as follows:

$$\begin{aligned}L_S^{*}=&\frac{|W_{\sigma\sigma}|}{2}\left(1-\alpha^{*}\right)\left[Var\left(K-\bar{\kappa}\right)+2Cov\left(K-\bar{\kappa};\bar{\kappa}-\bar{\kappa}^{*}\right)\right]+\\&+n\left(1-n\right)\frac{|W_{\sigma\sigma}|}{2}\left[Var\left(K_1-\kappa_1-\left(K_2-\kappa_2\right)\right)+\right]\\&+n\left(1-n\right)|W_{\sigma\sigma}|\,Cov\left(K_1-\kappa_1-\left(K_2-\kappa_2\right);\kappa_1-\kappa_1^{*}-\left(\kappa_2-\kappa_2^{*}\right)\right)\\&+\frac{|W_{\sigma\sigma}|}{2}\left(n\sigma_1^2+\left(1-n\right)\sigma_2^2\right)\end{aligned}\tag{35}$$

The first term in (35) represents the variance of the gap between the average equilibrium actions K and the average actions under complete information. The gap between the average actions under incomplete and incomplete information is defined by the errors in the public signals and can be written as follows:

$$K-\bar{\kappa}=n\kappa\left(\gamma_1+\alpha\left(1-n\right)\left(1-\gamma_1\right)\right)\left(z_1-\theta_1\right)+\left(1-n\right)\kappa\left(\gamma_2+\alpha n\left(1-\gamma_2\right)\right)\left(z_2-\theta_2\right),\tag{36}$$

where $(z_j-\theta_j)$ represents the error in the public information about the fundamental θ_j. Thus, the variance of the gap is defined by the variance of two public sets of information. As two fundamentals are uncorrelated, the variance of the gap is equal to the sum of two terms, each of which is defined by the variance of one set of information:

$$Var\left(K-\bar{\kappa}\right)=Var_1\left(K-\bar{\kappa}\right)+Var_2\left(K-\bar{\kappa}\right)$$

$$Var_j\left(K-\bar{\kappa}\right)=n_j^2\kappa^2\left(\gamma_j+\alpha\left(1-n_j\right)\left(1-\gamma_j\right)\right)^2\sigma_{z,j}^2$$

The gap between equilibrium and efficient average actions under complete information is defined as follows:

$$\bar{\kappa}-\bar{\kappa}^{*}=\left(\kappa-\kappa^{*}\right)\left(n\theta_1+\left(1-n\right)\theta_2\right)$$

If $\kappa=\kappa^{*}$, the average equilibrium actions and efficient average actions coincide under complete information. Thus, in this case the equilibrium is efficient on average. The covariance of this gap and the gap between the average actions under complete and incomplete information can be written as the sum of two terms, each of which depends on the precision of one set of information. The term which depends on the information about fundamental θ^j is as follows:

$$Cov_j\left(K-\bar{\kappa};\bar{\kappa}-\bar{\kappa}^*\right)=-n_j^2\kappa\left(\kappa-\kappa^*\right)\left(\gamma_j+\alpha\left(1-n_j\right)\left(1-\gamma_j\right)\right)\sigma_{z,j}^2,$$

where we use $Cov(z_j-\theta_j,\theta_j)=-\sigma_{z,j}^2$

The difference between the gaps in private actions is also defined by the errors in public information:

$$\left(K_1-\kappa_1-\left(K_2-\kappa_2\right)\right)=\kappa\left(\left(1-\alpha\right)\gamma_1-\alpha n\left(1-\gamma_1\right)\right)\left(z_1-\theta_1\right)-\kappa\left(\left(1-\alpha\right)\gamma_2-\alpha\left(1-n\right)\left(1-\gamma_2\right)\right)\left(z_2-\theta_2\right)$$

Thus, the variance of this variable is also separable into two terms. For example, the term which depends on the information about region j is as follows:

$$Var_j\left(K_1-\kappa_1-\left(K_2-\kappa_2\right)\right)=\kappa^2\left(\left(1-\alpha\right)\gamma_j-\alpha n_j\left(1-\gamma_j\right)\right)\sigma_{z,j}^2$$

The relative gap between regions in equilibrium and social optimum under complete information is defined as follows:

$$\kappa_1-\kappa_1^*-\left(\kappa_2-\kappa_2^*\right)=\left(1-\alpha\right)\kappa\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}\left(\theta_1-\theta_2\right) \tag{37}$$

As we can see, this gap is present only if $U_{\sigma\sigma}$ is different from zero, meaning that the agents value the dispersion in private actions (either positively or negatively). Covariance of two measures of the gap between the regions is separable into two terms with

$$\begin{aligned}&Cov_j\left(K_1-\kappa_1-\left(K_2-\kappa_2\right);\kappa_1-\kappa_1^*-\left(\kappa_2-\kappa_2^*\right)\right)=\\&=-\kappa^2\left(\left(1-\alpha\right)\gamma_j-\alpha n_j\left(1-\gamma_j\right)\right)\left(1-\alpha\right)\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}\sigma_{z,j}^2\end{aligned}$$

Finally, the variances of private actions are measured as follows:

$$\sigma_j^2=\left(1-\gamma_j\right)^2\left(1-\alpha\left(1-n_j\right)\right)^2\kappa^2\sigma_{x,j}^2$$

As the variance of private actions in any region depends only on the precisions of information about this region, we conclude that social loss is separable into two arguments:

$$L_S^*=L_S^1+L_S^2$$

where the term L_S^j is the component which depends on the information about region j:

$$L_S^j = \frac{|W_{\sigma\sigma}|}{2}(1-\alpha^*) n_j^2 \sigma_{z,j}^2 \left[\kappa^2 (\gamma_j + \alpha(1-n_j)(1-\gamma_j))^2 - 2\kappa(\kappa-\kappa^*)(\gamma_j + \alpha(1-n_j)(1-\gamma_j))\right] + \quad (38)$$
$$+ n_j (1-n_j) \frac{|W_{\sigma\sigma}|}{2} \kappa^2 \sigma_{z,j}^2 ((1-\alpha)\gamma_j - \alpha n_j (1-\gamma_j))^2 +$$
$$- n_j (1-n_j) |W_{\sigma\sigma}| \kappa^2 \sigma_{z,j}^2 ((1-\alpha)\gamma_j - \alpha n_j (1-\gamma_j)) (1-\alpha)\rho +$$
$$+ \frac{|W_{\sigma\sigma}|}{2} n_j (1-\gamma_j)^2 (1-\alpha(1-n_j))^2 \kappa^2 \sigma_{x,j}^2,$$

where $\rho = \frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$. We apply this general loss function to study the social welfare properties of information in the next sub-section.

4.4 Social value of information

Exploring the properties of social loss function (38) allows to study the social value of information in a segmented economy and to compare it with its value in a homogeneous economy. The properties of information in a homogeneous economy have been described in Angeletos and Pavan (2007), who come with three main findings:

- in efficient economies with $\kappa = \kappa^*$ and $\alpha = \alpha^*$, social loss is decreasing in the precision of both public and private information;
- in economies with efficient equilibrium allocation under complete information ($\kappa = \kappa^*$) and inefficient equilibrium degree of coordination ($\alpha \neq \alpha^*$), $\alpha^* > \alpha > 0$ suffices for social loss to be decreasing in the precision of public information and $\alpha^* < \alpha < 0$ suffices for social loss to be decreasing in the precision of private information;
- in inefficient economies with $\kappa \neq \kappa^*$, there exist $\overline{\phi}$ and $\underline{\phi}$ such that social loss is decreasing in precision of both public and private information, if $\kappa^* - \kappa > \overline{\phi}$, and increasing in precision of both public and private information, if $\kappa^* - \kappa < \underline{\phi}$.

We start our testing of these results in segmented economies under assumption that the dispersion in private action does not create any externality ($U_{\sigma\sigma} = 0$). This allows us to abstract from the source of inefficiency which is present in a segmented economy, but does not affect social welfare in homogeneous economy. The findings about the social value in such economy are summarized in the following Proposition:

Proposition 4. ***The social value of information in economies without externality created by the dispersion in private actions.*** *In segmented economies with $U_{\sigma\sigma} = 0$ the social loss function is such that:*

1. *in economies with $\kappa = \kappa^*$ and $\alpha = \alpha^*$, social loss is decreasing in precision of private and public information;*
2. *in economies with $\kappa = \kappa^*$ and $\alpha \neq \alpha^*$, $\alpha^* > \alpha > 0$ is sufficient condition for social loss to decrease in precision of public information and $\alpha - \psi < \alpha^* < \alpha < 0$ with $\psi = -\frac{(1-\alpha)(1-\alpha n)}{2\alpha n(1-n)} > 0$ is sufficient condition for social loss to decrease in precision of private information;*
3. *in economies with $\kappa \neq \kappa^*$, for any (α, α^*, n) there exist $\overline{\phi}$ and $\underline{\phi}$ such that*
   (a) *if $\alpha > 0$, social loss is decreasing in precision of public and private information if $\kappa^* - \kappa > \overline{\phi}(\alpha, \alpha^*, n)$ and increasing in precision of public and private information if $\kappa^* - \kappa < \underline{\phi}(\alpha, \alpha^*, n)$;*

(b) *if $\alpha < 0$, social loss is decreasing in precision of private information if $\kappa^* - \kappa > \overline{\phi}(\alpha, \alpha^*, n)$ and increasing in precision of private information if $\kappa^* - \kappa < \underline{\phi}(\alpha, \alpha^*, n)$.*

Part 1 of Proposition 4 shows that the social value of both private and public information is positive in segmented efficient economies. This finding corresponds to the value of information in efficient homogeneous economies. Part 2 of Proposition 4 implies that the sufficient condition for public information to be valuable is the same in segmented and homogeneous economies, if private actions are characterized by strategic complementarity. Similar to Angeletos and Pavan (2007), the positive gap between the efficient and the equilibrium degree of coordination ensures that the social loss is decreasing in the precision of public information.

Nevertheless, the sufficient condition for private information to be welfare-improving is now different. As we can see in Part 2 of Proposition 4, the social loss is necessarily increasing in the precision of public information is $\alpha^* \in (\alpha - \psi, \alpha)$. This means that for a large gap between the efficient and the equilibrium degree of substitutability, the social value of private information may be negative. The reasoning is straightforward. With the help of equation (35), we can show that the loss in economy with inefficient degree of coordination is equal to the loss in efficient economy plus the loss created by inefficient degree of substitutability:

$$L_S^* = L_S^*|_{\alpha=\alpha^*} + (\alpha - \alpha^*) \frac{|W_{\sigma\sigma}|}{2} Var(K - \bar{\kappa}),$$

where $Var(K - \bar{\kappa})$ is the variance of the gap between the average equilibrium actions under complete and incomplete information. As we have seen earlier in equation (36)), this gap is proportional to $(\gamma_j + \alpha(1 - n_j)(1 - \gamma_j))$, which stands for the normalized weight of public and private information in private actions. In homogeneous economy with $n_j = 1$, this weight is equal to the relative weight of home public information in private actions, γ_j. Thus, in a homogeneous economy this weight is positive. An increase in the precision of private information leads to a decrease in the relative weight of home public information. This leads to a lower impact of the errors in the home public information on the average actions and lower dispersion $Var(K - \bar{\kappa})$. In a two-region economy the value $(\gamma_j + \alpha(1 - n_j)(1 - \gamma_j))$ may be negative for sufficiently strong strategic substitutability in private actions. The negative gap between the average equilibrium actions under complete and incomplete information means that the average actions are too high in equilibrium if the value of public signal is too low in comparison with the real value of the fundamentals. This phenomenon arises because the agent tries to keep their actions apart from the actions of others not only in their home region, but also from the actions of foreigners. Because of the negative value of the gap, an increase in the precision of private information and a decrease in the relative weight of public signal lead to an increase, not decrease, in the absolute value of this gap. This implies an increase in the precision of private information together with the high value of $(\alpha - \alpha^*)$ may cause an increase in social loss.

Equivalent reasoning explains, why Part 3 of Proposition 4 differs from its analogue in a homogeneous economy. As we can see, the findings about the value of information in inefficient economies with strategic substitutability are different. The social loss in these economies is equal to the loss in economies with efficient equilibrium allocation under complete information plus the loss created by inefficiency in complete-information equilibrium:

$$L_S^* = L_S^*|_{\kappa=\kappa^*} + (1 - \alpha^*) |W_{\sigma\sigma}| Cov(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*),$$

where $Cov_j(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*)$ may negatively depend on the precision of public information in case of strategic substitutability and low relative precision of public information. The main differences in social value of information in segmented economies in comparison with the homogeneous economies are summarized by the following Corollary:

Corollary 2. *In segmented economies with strategic substitutability $\alpha < 0$ and $\rho = 0$, contrary to the corresponding homogeneous economies,*

1. *if $\kappa = \kappa^*$ and $\alpha^* < \alpha - \psi < \alpha < 0$, private information may be detrimental for social welfare;*
2. *social loss may be increasing in precision of public information, if $\kappa^* - \kappa$ is sufficiently high, and decreasing in precision of public information, if $\kappa^* - \kappa$ is sufficiently low.*

The presence of externality created by the dispersion in private actions, meaning that $U_{\sigma\sigma} \neq 0$, affects the social loss trough the gap in actions between regions:

$$L_S^* = L_S^*|_{U_{\sigma\sigma}=0} + n(1-n)|W_{\sigma\sigma}|\,Cov\left(K_1 - \kappa_1 - (K_2 - \kappa_2);\kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*)\right),$$

where $\kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*) = (1-2n)(1-\alpha)\kappa\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}(\theta_1 - \theta_2)$ and $Cov_j\left(K_1 - \kappa_1 - (K_2 - \kappa_2);\kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*)\right) = -\kappa^2\left((1-\alpha)\gamma_j - \alpha n_j(1-\gamma_j)\right)(1-\alpha)\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}\sigma_{z,j}^2$. In case of strategic complementarity, term $\left((1-\alpha)\gamma_j - \alpha n_j(1-\gamma_j)\right)\sigma_{z,j}^2$ is increasing in the precision of public information. Thus, the negative value of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ ensures that covariance of the two inter-regional gaps is increasing. In case of strategic substitutability, the value of this term may be decreasing in the precision of public information for high values of ζ_j. Thus, the positive value of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ is required for social loss to be increasing in the precision of public information. Value $\left((1-\alpha)\gamma_j - \alpha n_j(1-\gamma_j)\right)$ is decreasing in the precision of private information for both strategic complementarity and substitutability, thus relatively high value of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ leads to a negative value of private information. As we have seen earlier, the positive gap between efficient and equilibrium allocation under complete information leads to an increase in the social value of information. Thus, more extreme values of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ are needed we retain the increasing social loss function. These findings are summarized in the following Proposition:

Proposition 5. ***The social value of information in economies with** $U_{\sigma\sigma} \neq 0$.* *For given $(\kappa, \kappa^*, \alpha, \alpha^*, n)$, there exist $0 < \overline{\rho} < 1$, $\underline{\rho} < 0$ and $\tilde{\rho} < 1$, such that*

1. *social loss is increasing in precision of public information if $\alpha > 0$ and $\rho < \underline{\rho}(\kappa, \kappa^*, \alpha, \alpha^*, n)$ or if $\alpha < 0$ and $\rho > \overline{\rho}(\kappa, \kappa^*, \alpha, \alpha^*, n)$, at least for low values of ζ;*
2. *social loss is increasing in precision of private information if and only if $\rho > \tilde{\rho}(\kappa, \kappa^*, \alpha, \alpha^*, n)$;*
3. *an increase in the gap $\kappa^* - \kappa$ and $\alpha^* - \alpha$ leads to an increase in $\overline{\rho}(\kappa, \kappa^*, \alpha, \alpha^*, n)$ and $\tilde{\rho}(\kappa, \kappa^*, \alpha, \alpha^*, n)$ and to a decrease in $\underline{\rho}(\kappa, \kappa^*, \alpha, \alpha^*, n)$.*

All these results are closely related to the relative influence of strategic private motive and the externality created by the dispersion in private actions on the social loss. In case of strategic complementarity, public information is more likely to have the positive value for the social and private welfare. If it is accompanied by negative externality of the dispersion in private actions ($U_{\sigma\sigma} < 0$ and $\rho > 0$), the public information becomes even more desirable, as the social planner wants to avoid any dispersion in private actions. Sufficiently high positive value of ρ, meaning that the negative externality of the dispersion is substantial, ensures that the social loss is decreasing in the precision of public information, even if the efficient degree of coordination is lower than the equilibrium degree. On the contrary, substantial positive externality ($U_{\sigma\sigma} > 0$ and $\rho < 0$) forces the social planner to look for greater dispersion in private actions, despite of strategic complementarity. This suffices for public information to be undesirable. In case of strategic substitutability, the agents use the public signals not only to predict their home fundamental, but also to differentiate their actions from the foreign private actions. Thus, an increase in the precision of public information leads to an increase in inter-regional dispersion. If the social loss of dispersion is sufficiently high ($U_{\sigma\sigma} < 0$ and $\rho > 0$), this may lead to a decrease in the social welfare, making the social value of public information negative. Moreover, sufficiently strong negative value of dispersion assures that the social value of private information may be negative, despite the type of strategic effect in private actions.

Thus, we discussed the welfare properties of information in segmented economies. In the next section we proceed to the discussion of the regional welfare effects of public and private information.

5 Regional welfare analysis

In this section we discuss the regional welfare properties of information. For this purpose we describe the regionally optimal allocations under complete and incomplete information. After that we derive the regional loss function, which is then used to study the effects of public and private information on the welfare of each region.

The regional welfare is the sum of private payoffs inside the region j:

$$W^j \equiv \int_{i \in S^j} U\left(k_i^j\left(x_i^j, z^j, z^{-j}\right), K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \theta^j\right) \mathrm{d}i$$

Similar to the social welfare studied in the previous section, the regional welfare consists of two terms:

$$W^j = \omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right) + \frac{\left(W_{\sigma\sigma}^j\right)^T}{2}\begin{bmatrix}\sigma_{k,j}^2 \\ \sigma_{k,j}^2\end{bmatrix}, \tag{39}$$

where $\omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right)$ is the regional welfare component, which depends on the average actions and the fundamental shocks:

$$\omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right) = n_j U\left(K_j, K, 0, \theta^1\right) + \frac{U_{\sigma\sigma}}{2} n_j^2\left(1 - n_j\right)\left(K_j - K_{-j}\right)^2 \tag{40}$$

According to (40), the regional welfare depends positively on the gap between regions, if the private value of dispersion $U_{\sigma\sigma}$ is positive. If private value of dispersion is negative, the regional welfare depends negatively on the gap between regions. The sign of this dependence coincides with the effect of the gap between regions on the social welfare. Nevertheless, the size of this effect is different. According to (23), the importance of this gap relative to the average payoff in the region is equal to $(1 - n_j)$. According to (40), the importance of the gap relative to the average regional payoff is equal to $n_j\left(1 - n_j\right)$. Thus, the regional society pays less attention to the gap between regions than the social planner.

The second term in regional welfare (39) demonstrates the regional and international values of dispersion inside the regions. Vector $W_{\sigma\sigma}^j$ is as follows:

$$W_{\sigma\sigma}^j = \begin{bmatrix} n_j\left(n_j U_{\sigma\sigma} + U_{kk}\right) \\ n_j\left(1 - n_j\right) U_{\sigma\sigma}\end{bmatrix} \tag{41}$$

The first element of this vector demonstrates the regional value of the dispersion in private actions in the home region. Under assumptions made at the beginning, this value is negative. Thus, the regional society does not like the variance in its home actions. Nevertheless, the absolute value of aversion to the home dispersion differs from the social aversion. In the previous section we have seen that the social aversion to dispersion in region j is equal to $n_j\left(U_{\sigma\sigma} + U_{kk}\right)$. The absolute value of the aversion is lower than the local aversion if $U_{\sigma\sigma}$ is positive. If $U_{\sigma\sigma}$ is negative, the local aversion to dispersion is lower that the social aversion.

The second element of vector $W^j_{\sigma\sigma}$ demonstrates the inter-regional value of dispersion. If private value of dispersion is positive, the local society gets welfare gains from the dispersion in actions abroad. If the private value of dispersion is positive, the local society gets a welfare loss from the dispersion abroad. Thus, the inter-regional value of dispersion may differ from the social value of dispersion which is always negative. The regional planner would like to impose the infinite noise in the actions in the other region. Nevertheless, we assume that this is not possible and the regional planner cannot discriminate between private agents in the foreign region. This assumption does not change the conclusions about the regional and inter-regional value of information which are studied in the subsequent sections.

5.1 Regional optimum under complete information

Under assumption of impossibility to discriminate between private agents, the regionally efficient allocation is the solution of the program of the regional planner. This allocation is a pair of strategies $\left(\tilde{\kappa}^j_j, \tilde{\kappa}^j_{-j}\right)$: $\mathbb{R}^2 \to \mathbb{R}^2$ such that

$$\left\{\tilde{\kappa}^j_j\left(\Theta^j\right), \tilde{\kappa}^j_{-j}\left(\Theta^j\right)\right\} = \arg\max_{\{K_j, K_{-j}\}} \omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right),$$

Thus, the regional planner chooses two strategies which maximize the component of its welfare, which does not depend on the dispersion. This representation is correct, as the planner does not value the dispersion inside its home region and chooses the same actions for all the agents in its region. Potentially, the regional planner would value the dispersion abroad, but the assumption made before does not allow the discrimination between the foreign agents. In this case the regionally optimal strategies are as follows:

$$\tilde{\kappa}^j_l\left(\theta^j, \theta^{-j}\right) = \tilde{\kappa}^j_{l,j}\theta^j,\ l \in \{j, -j\} \tag{42}$$

with

$$\tilde{\kappa}^j_{j,j} = \kappa^* - (1-n)\,\tilde{\kappa}_{jj} \tag{43}$$

$$\tilde{\kappa}^j_{-j,j} = \kappa^* - \tilde{\kappa}_{-jj}, \tag{44}$$

where $\tilde{\kappa}_{jj} = -\frac{(U_{kK}+U_{KK})((U_{kK}+U_{KK})U_{k\theta}+(U_{kk}+U_{kK})U_{K\theta})}{W_{KK}\left[(1-n)\left(U^2_{kK}-U_{kk}U_{kK}\right)-nW_{KK}U_{\sigma\sigma}\right]}$ and $\tilde{\kappa}_{-j,j} = \frac{(U_{kk}+U_{kK}+n(U_{kK}+U_{KK}))((U_{kK}+U_{KK})U_{k\theta}-(U_{kk}+U_{kK})U_{K\theta})}{W_{KK}\left[(1-n)\left(U^2_{kK}-U_{kk}U_{kK}\right)-nW_{KK}U_{\sigma\sigma}\right]}$

The regional optimum for region j implies that agents do not react to the shocks in region $-j$. The reason for this is that the private payoffs in region j depend only on the fundamentals in region j. Consequently, the regional planner wants all agents in the economy to base their actions on the fundamentals in region j, irrespective of the place where agents live. In equilibrium, at least the agents in region $-j$ do react to the shocks in their region. Thus, the following Corollary states the impossibility of equilibrium to be regionally optimal:

Corollary 3. *As far as $U_{k\theta} \neq 0$, the equilibrium is not regionally optimal.*

Moreover, the presence of externalities makes the regionally efficient allocation not optimal from the social point of view. As we have seen earlier, the regional planner do not take into account the average payoff in the other region. The relative importance of the gap between regions is lower for the regional planners than for the social planner. The absolute value of the regional aversion to the regional dispersion does not coincide with the social aversion. The sign of aversion to the foreign dispersion may be opposite to the social one. All this implies that in general, the regional optimum is not socially efficient. The direct consequence of this is the possible inefficiency of information policies if they are developed regionally.

5.2 Regional optimum with incomplete information

The regionally efficient allocation under incomplete information with assumption that the regional planner cannot discriminate between agents is achieved with a pair of strategies $\left(\tilde{k}_j^j, \tilde{k}_{-j}^j\right)$: $\mathbb{R}^3 \to \mathbb{R}^2$ such that

$$\left\{\tilde{k}_j^j\left(x^j, Z\right), \tilde{k}_{-j}^j\left(x^{-j}, Z\right)\right\} = \arg\max_{k'(x,Z)} E\left[W^j\left(k\left(x, Z\right), K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \Theta\right)\right], \tag{45}$$

First-order condition for this problem is similar to the first-order condition of the problem of social planner:

$$\begin{aligned}\tilde{k}_l^j\left(x^l, z^j, z^{-j}\right) = {} & E\left[\tilde{\kappa}_l^j\left(\Theta\right) + \tilde{\alpha}_{l,j}\left(K^j\left(\Theta, Z\right) - \tilde{\kappa}_j^j\left(\Theta\right)\right)\Big| x^l, z^j, z^{-j}\right] \\ & + E\left[\tilde{\alpha}_{l,-j}\left(K^{-j}\left(\Theta, Z\right) - \tilde{\kappa}_{-j}^j\left(\Theta\right)\right)\Big| x^l, z^j, z^{-j}\right] \quad l \in \{j, -j\},\end{aligned} \tag{46}$$

where $\tilde{\alpha}_{j,j} = n\frac{n(U_{\sigma\sigma}-U_{KK})-2U_{kK}}{nU_{\sigma\sigma}+U_{kk}}$ is the regionally optimal coordination inside region j and $\tilde{\alpha}_{j,-j} = (1-n)\frac{n(U_{\sigma\sigma}-U_{KK})-U_{kK}}{nU_{\sigma\sigma}+U_{kk}}$ is regionally optimal coordination between the regions. This first-order condition gives the following regionally optimal linear strategy for agents in the home region of the regional planner:

$$\tilde{k}_j^j\left(x^j, z^j, z^{-j}\right) = \tilde{\kappa}_{j,j}\left(\tilde{\gamma}_j z^j + \left(1-\tilde{\gamma}_j\right)x^j\right) \tag{47}$$

where

$$\tilde{\gamma}_j = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\tilde{\alpha}_{j,j}}{1-\left(1-\delta^j\right)\tilde{\alpha}_{j,j}} + \frac{\left(1-\delta^j\right)\tilde{\alpha}_{j,-j}}{1-\left(1-\delta^j\right)\tilde{\alpha}_{j,j}}\frac{\tilde{\kappa}_{-j,j}}{\tilde{\kappa}_{j,j}} \tag{48}$$

Thus, in regionally efficient distribution the agents in the home region weight their private and public information about their home fundamentals and ignore the information about foreign shocks. The actions in the foreign region $-j$, if chosen by the planner in home region j, rely only on public information about the home region z^j:

$$\tilde{k}_{-j}^j\left(x^{-j}, z^j, z^{-j}\right) = \tilde{\kappa}_{-j,j}z^j \tag{49}$$

These strategies are incompatible with neither the equilibrium nor the social optimum. Thus, the information policy may be inefficient if chosen by the local authority. For example, if $U_{\sigma\sigma} > 0$, the regional planner in region j would like to impose the variance to the private actions in region $-j$ and to stretch the gap between regions. In the equilibrium, the regional authority cannot influence the private dispersion abroad, as it is defined only by the information about the foreign regional fundamental, as expression (21) clarifies. Nevertheless, it can impose additional noise to its signal about the regional fundamental to stretch the gap given by (20). This can increase the gap between the equilibrium actions in foreign region and the optimal actions, but the planner does not take this external effect into account. Thus, it would choose excessively opaque policy without publishing precise information about its region fundamentals. We will discuss the difference between the local and global welfare criteria in the next section after discussing the regional loss functions.

5.3 Regional loss functions

Similar to the social welfare in the previous section, the expected regional welfare can be rewritten as follows:

$$EW^j = \omega^j\left(\tilde{\kappa}_1^j, \tilde{\kappa}_2^j, \theta^1, \theta^2\right) - \mathscr{L},$$

where $\omega^j\left(\tilde{\kappa}_1^j, \tilde{\kappa}_2^j, \theta^1, \theta^2\right)$ is the value of regional welfare under the regionally optimal distribution and $\mathscr{L}_j$ is the social loss which arises due to the gap between the equilibrium and the regional optimum. The value of this loss is as follows:

$$\mathscr{L}_j = n_j\frac{|W_{KK}|}{2}Var\left(K-\tilde{K}^j\right) + n_j\left(1-n_j\right)|U_{kk}|\left(1-\alpha\right)Cov\left(K-\tilde{K}; K_j-\tilde{\kappa}_j^j-\left(K_{-j}-\tilde{\kappa}_{-j}^j\right)\right) \tag{50}$$
$$-n\left(1-n\right)\frac{\left(1-n\right)U_{kk}+nU_{\sigma\sigma}}{2}Var\left(K_j-\tilde{\kappa}_j^j-\left(K_{-j}-\tilde{\kappa}_{-j}^j\right)\right)+n\frac{|U_{kk}+nU_{\sigma\sigma}|}{2}\sigma_{k,j}^2-n\left(1-n\right)\frac{U_{\sigma\sigma}}{2}\sigma_{k,-j}^2$$

Thus, equation (50) reveals the sources of inefficiency from the regional perspective. The first source of inefficiency is the gap between the equilibrium average actions and the regional efficient average actions $\tilde{K}^j$. The variance of this gap is denoted by $Var\left(K-\tilde{K}^j\right)$. The second source of regional inefficiency is the covariance between the average gap $K-\tilde{K}$ and the relative gap between regions $K_j-\tilde{\kappa}_j^j-\left(K_{-j}-\tilde{\kappa}_{-j}^j\right)$. The variance of this relative gap is the third source of inefficiency from the regional perspective. The last two sources of regional loss are the dispersion of private actions in both regions. The variance in the home private actions σ_j^2 increases the regional loss. The variance in the foreign private actions σ_{-j}^2 increases the regional loss only if the private value of dispersion is negative ($U_{\sigma\sigma}<0$). If the private value of dispersion is positive ($U_{\sigma\sigma}>0$), the regional loss depends negatively on the dispersion in private actions in the other region.

Similar to the previous section, the regional loss can be rewritten as a sum of component L_j^0, which is independent of information structure, and component L_j, which depends on the information structure:

$$\mathscr{L}_j = L_j^0 + L_j$$

As expression L_j is rather massive, it is given in Appendix C1. Due to the absence of correlation between the regional sources of information, the loss component L_j can be represented as a sum of two components, each of which depends on the information about one of the regions:

$$L_j = L_{j,j} + L_{j,-j},$$

where $L_{j,j}$ is the regional loss in region j, caused by the incompleteness of information about region j and $L_{j,-j}$ is the loss in region j, caused by the incompleteness of information about region $-j$. Thus, $L_{j,j}$ characterizes the regional value of information about region j, while value $L_{j,-j}$ characterizes the inter-regional value of information about region $-j$.

Obviously, the social loss L_S^j, which measures the loss in social welfare because of the incompleteness of information about fundamental θ^j, is the sum of the losses in two regions:

$$L_S^j = L_{j,j} + L_{-j,j}.$$

Component $L_{-j,j}$ measures the side-effects of the information about region j suffered by region $-j$. In the next section we study the regional and inter-regional value of information in the general model.

5.4 Regional and inter-regional value of information

The regional loss function (50) reveals the importance of parameter $U_{\sigma\sigma}$ for the regional value of public and private information. Thus, we start with the efficient economy without the externality created by the dispersion in private actions ($U_{\sigma\sigma} = 0$). After that we discuss the regional and inter-regional effects of information in economies, where the only source of inefficiency is the dispersion in private actions ($\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} \neq 0$). We conclude with the general model, where all sources of inefficiency may be present.

The regional and inter-regional welfare effects of information in globally efficient economies are presented by the following proposition:

Proposition 6. ***Regional and inter-regional value of information in globally efficient segmented economies.*** *In economies with $\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} = 0$ for given (α, n_j), there exist $\bar{\zeta}_{j,H} \geq 0$ and $\bar{\zeta}_{j,F} \geq 0$ such that*

1. *if $\alpha > 0$, the regional loss is decreasing in precision of both public and private home information;*
2. *if $\alpha < 0$, the regional loss is decreasing in precision of public home information, but is increasing in the precision of home private information, if $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,H}(\alpha, n_j)$. If $1 - \alpha n_j(2n_j - 1) > 0$, threshold $\bar{\zeta}_{j,H}(\alpha, n_j) = 0$;*
3. *the regional loss is increasing in foreign private information precision, if $\alpha > 0$, and decreasing in foreign private information precision, if $\alpha < 0$;*
4. *if $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,F}(\alpha, n_j)$, the regional loss is increasing in the precision of foreign public information, if $\alpha > 0$, and decreasing in the precision of foreign public information, if $\alpha < 0$. If $\alpha\left(1 - 2n_j + \alpha n_j^2\right) > 0$, threshold $\bar{\zeta}_{j,F}(\alpha, n_j) = 0$.*

Part 1 of Proposition 6 states that the regional value of both home private and public information is positive, if there is strategic complementarity in private actions. If we compare this result with the social value of information in Proposition 4, we will see that in this case the social value of information coincides with the regional value of information. This means that if the informational policy was delegated to the regional authority, it would be socially optimal. Such a social authority would try to achieve the highest possible precision of both public and private information. This is not the case in economies with strategic substitutability.

Part 2 of Proposition 6 indicates, that the regional value of private information may be negative. This happens, if the regional size is relatively small ($n_j < 1/2$), strategic substitutability is sufficiently strong ($\alpha < -n_j^{-1}(1 - 2n_j)^{-1}$) and the relative precision of public information is sufficiently low ($\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,H}(\alpha, n_j)$). The intuition is as follows. An increase in the precision of home private information forces private agents to increase the weight of this information in their actions. Together with the strong strategic substitutability, this may lead to an increase in the dispersion of private actions and the dispersion of the relative gap between regions, which is detrimental for the regional welfare, according to (50). The effect of the dispersion in the gap between regions depends negatively on the region size. This gives Part 2 of Proposition 6. The negative regional value of private information means that the local authority may choose globally inefficient information structure. For strong strategic substitutability, the local authority may have the incentive to restrict the possible precision of private information.

Part 3 and 4 of Proposition 6 describe the inter-regional value of information. The inter-regional value of information represents the effect of information about region j on the regional welfare of region $-j$. Proposition 6 states that the inter-regional value of information depends on the region size, the equilibrium degree of coordination and on the relative precision of public information $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2}$.

According to Part 3 of Proposition 6, the inter-regional value of private information is positive, if there is strategic substitutability. In this case the agents value the coordination negatively. Thus, an increase in the precision of private information leads to an increase in its weight in the home private actions. As a result, agents in region j rely less on their home public information and the coordination between regions becomes smaller, which increases the private payoffs in region $-j$. If there is strategic complementarity, private information has negative inter-regional value. As we have already seen, an increase in the precision of private information in region j lowers the inter-regional coordination, which is undesirable for the agents in the other region.

According to Part 4 of Proposition 6, an increase in the precision of public information about region j leads to an increase in the regional loss in region $-j$ in case of strong strategic complementarity, such that α is positive and higher than $\frac{2n-1}{n^2}$. This happens because an increase in precision of foreign public information pushes the private actions in region $-j$ away from the relevant fundamental shock. Thus, the inter-regional effect of higher precision of public information is negative. If strategic complementarity is not so strong and $0 < \alpha < \frac{2n_j-1}{n_j^2}$, the inter-regional loss may decrease in the precision of public information up to some limit. Thus, if the value of public information precision is limited for some technological reason, there may be positive inter-regional value. Worth to note that this is possible only if the size of region j is larger than $1/2$. Otherwise, the externality created by its information is not enough to reverse its effect on the other region.

In case of strong strategic substitutability, such that α is negative and lower than $\frac{2n-1}{n^2}$, the inter-regional loss is positive, meaning that there is a negative inter-regional externality. Nevertheless, an increase in the precision of public information in region j lowers the loss in region $-j$. More precise public information about the foreign fundamentals helps private agents to better predict the foreign actions and to keep their own actions away from coordination between regions. Thus, the inter-regional value of public information is positive. If strategic substitutability is modest and $\frac{2n_j-1}{n_j^2} < \alpha < 0$, inter-regional loss is non-monotonic function of the precision of public signal. For low relative precision of public information, its increase may have negative inter-regional value. Nevertheless, this phenomenon takes place only if the size of region j is lower than $1/2$.

The presence of externality created by the dispersion in private actions may change considerably the regional and inter-regional effects of information. These effects are summarized in the following proposition:

Proposition 7. ***Regional and inter-regional value of information in economies, which are efficient if population is not segmented and inefficient with segmented population.*** *In economies with $\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} \neq 0$, for given (α, n_j) there exist $\underline{\rho}_j < 0 < \overline{\rho}_j$, $\tilde{\rho}_j$ and $\bar{\zeta}_j, \bar{\zeta}_{j,H} \geq 0$ such that:*

1. *if $\alpha > 0$ and $\rho < \underline{\rho}_j(\alpha, n_j)$ or $\alpha < 0$ and $\rho > \bar{\rho}_j(\alpha, n_j)$, the regional loss is increasing in the precision of home public information for all $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_j(\alpha, n_j)$;*
2. *the regional loss is increasing in the precision of home private information, if $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,H}(\alpha, n_j)$, $\alpha < 0$ and $\rho > \tilde{\tilde{\rho}}_j(\alpha, n_j)$;*
3. *the regional loss is increasing in precision of foreign public and private information, if $\rho > \tilde{\rho}_j(\alpha, n_j)$.*

Proposition 7 shows that the welfare properties of the dispersion in private actions changes the regional and inter-regional value of information. Part 1 of the proposition indicates that strategic complementarity accompanied with a large positive value of dispersion ($\rho < \underline{\rho}_j(\alpha, n_j) < 0$) makes the regional value of public information negative. If $U_{\sigma\sigma}$ is positive, the regional loss depends positively on the average gap in actions between the regions. Moreover, the negative impact of the home dispersion in actions on the regional welfare is not that large. Thus, the region may be better off with the smaller precision of home public information, despite the strategic complementarity in private actions. On the contrary, sufficiently large positive value of ρ is necessary for the regional value of public information to be negative, if there is strategic substitutability.

Part 2 of Proposition 7 shows that the regional value of home private information is basically the same as it is in efficient economies, described in Proposition 6. The regional value of home private information is still negative in economies with strong strategic substitutability, if ρ is not too small. The large negative value of ρ means the large positive value of dispersion inside and between regions, that would overcome the effect of strategic substitutability on the value of private information. Thus, the regional value of private information is necessarily positive for $\rho < \tilde{\tilde{\rho}}_j(\alpha, n_j) < 0$.

Part 3 of Proposition 7 demonstrates that the inter-regional value of both public and private information is negative, if the dispersion inside and between regions has sufficiently strong negative effect on the regional welfare. Almost all these results holds in economies with socially inefficient degree of coordination. Regional and inter-regional welfare properties of information in these economies are summarized in the following Proposition:

Proposition 8. ***Regional and inter-regional value of information in economies with inefficient degree of coordination.*** *For given (α, α^*, n_j), there exist $\underline{\hat{\rho}} < 0 < \hat{\hat{\rho}}$, $\hat{\hat{\rho}}$, $\hat{\psi} > 0$ and $\bar{\zeta}_j \geq 0$ such that:*

1. *if $\alpha > 0$ and $\rho < \underline{\hat{\rho}}(\alpha, \alpha^*, n_j)$ or $\alpha < 0$ and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$, the regional loss is increasing in precision of home public information for all $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_j$. Thresholds $\underline{\hat{\rho}}(\alpha, \alpha^*, n_j)$ and $\hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$ depend positively on the gap $\alpha^* - \alpha$;*

2. *the regional loss is increasing in the precision of foreign private information, if $\alpha^* - \alpha < \hat{\psi}(\alpha, \alpha^*, n_j)$and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$, and increasing in precision of home private information, if $\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$ and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$;*

3. *if $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$, the regional loss is increasing in precision of foreign public information if $\alpha > 0$, $\alpha^* - \alpha < \hat{\psi}(\alpha, \alpha^*, n_j)$ and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$ or if $\alpha < 0$, $\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$.*

Part 1 of Proposition 8 shows that the regional welfare properties of public information coincide with its properties in economies with efficient degree of coordination. Nevertheless, the positive gap between the efficient and the equilibrium degree of coordination enlarges the region of values of ρ, for which the regional value of public information is negative under strategic complementarity. At the same time, it shrinks the region of values of ρ, for which the regional value of public information is negative under strategic substitutability. Higher α^* means lower impact of the volatility of the gap between the average equilibrium and regional efficient actions in the regional loss (50). This makes public information less valuable, if there is strategic complementarity and more valuable, if there is strategic substitutability.

Part 2 of Proposition 8 demonstrates that the regional value of private information may be negative, even if there is strategic complementarity in private actions. Sufficient positive gap between the efficient and equilibrium degree of coordination ($\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$) and sufficiently negative private value of dispersion ($\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$) suffices for it. Part 3 of Proposition 8 demonstrates that relatively large gap between the efficient and equilibrium degree of coordination ($\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$) suffices for foreign public information to be regionally desirable in case of strategic complementarity. The relatively large negative gap suffices for foreign public information to be regionally desirable in case of strategic substitutability.

In the next section we illustrate the social and regional welfare properties of information by a number of examples.

6 Applications

In this section we apply the social and regional welfare analysis to several examples. We start with two examples of the efficient economies. The first example illustrates the efficient competitive economy with strategic complementarity, the second refers to the efficient Lucas-Phelps island economy. After that we provide two examples of beauty-contest models with inefficient degree of coordination. One of this examples assumes that the dispersion in private action does not create externalities (Hellwig and Veldkamp (2009)), while the second implies the positive externality of the dispersion (Morris and Shin (2002) beauty contest).

6.1 Efficient economies

Efficient competitive economy

Two regions are inhabited by a continuum of households, which consume two goods. Initially, each household has an endowment w of good 2; good 1 should be produced. Each household is a producer and a consumer at the same time. Utility of agent i living in region j is given by the following function:

$$u_i^j = \nu\left(q_{1,i}^j\right) + q_{2,i}^j, \tag{51}$$

where $q_{1,i}^j$ and $q_{2,i}^j$ denote the consumed quantity of two goods, $\nu\left(q_{1,i}^j, \theta^j\right) = Aq_{1,i}^j - {}^b\!/_2\left(q_{1,i}^j\right)^2$, $b > 0$. Goods are sold in the common market at price p, which is the same for agents in both region.

The budget constraint for the household is

$$pq_{1,i}^j + q_{2,i}^j = w + \pi_i^j, \tag{52}$$

where price of good 2 is normalized to unity and π_i^j is the profit of agent i:

$$\pi_i^j = pk_i^j - C\left(k_i^j\right), \tag{53}$$

where k_i^j is the quantity of good 1 produced by agent i in region j and

$$C\left(k_i^j\right) = \frac{\left(k_i^j\right)^2}{2} - \theta^j k_i^j \tag{54}$$

is the cost of producing good 1. Parameter θ^j is region-specific technology shock. An increase in θ^j means that the first good becomes cheaper to produce for agents in region j. Maximization of utility (51) under budget constraint (52) gives the demand of the agent i, living in region j, for good 1:

$$q_{1,i}^j = \frac{A-p}{b} \tag{55}$$

As we can see from demand function (55), all agents consume the same quantity of good 1. The market-clearing condition is $bK = A - p$, where K is the total quantity of good 1, produced in the economy. This gives market-clearing price $p = A - bK$. The quantity of good 1 purchased by any agent at this price, is equal to K. The quantity of good 2 consumed by agent i is equal to $w + \pi_i^j - (A - bK)\,K$. As the profit of the household is equal to $\pi_i^j = pk_i^j - C\left(k_i^j\right) = (A - bK)\,k_i^j - \frac{\left(k_i^j\right)^2}{2} - \theta^j k_i^j$, we get the following utility of the household:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = \left(A + \theta^j - bK\right) k_i^j - \frac{\left(k_i^j\right)^2}{2} + b\frac{K^2}{2} + AK + w \quad (56)$$

It is easy to show that $U_{kk} = -1$, $U_{kK} = -b$, $U_{KK} = b$, $U_{k\theta} = 1$, $U_{K\theta} = U_{\sigma\sigma} = 0$. This implies that $\kappa = \kappa^* = \frac{1}{1+b}$, $\alpha = \alpha^* = -b$ and $\rho = 0$. In other words, the example describes the efficient economy with strategic substitutability in private actions, as $b > 0$ by assumption.[2]

One-region version of a similar efficient economy is derived in Angeletos and Pavan (2007), who show that the social value of both public and private information is positive. The welfare properties of information in this two-region model are listed in the following Corollary:

Corollary 4. *In efficient competitive economy with strategic substitutability described here,*

1. *social value of public and private information, regional value of public information and inter-regional value of private information are positive;*

2. *regional value of private information may be negative if $n_j < \frac{1}{2}$ and $b > \frac{1}{n_j(2n_j-1)}$;*

3. *inter-regional value of public information about cost shock in region j may be negative if $n_j < 1/2$ and $b < \frac{1-2n_j}{n_j^2}$.*

These findings are in line with Propositions 6 and 4. The regional value of private home information may be negative in small regions with strong strategic substitutability. On the contrary, the inter-regional value of public information may be negative for weak degree of strategic substitutability.

Lucas-Phelps island economy

Myatt and Wallace (2014) show that the preference of an agent in a Lucas-Phelps island economy can be described by the following utility function:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = \overline{u} - r\left(k_i^j - K\right)^2 - (1-r)\left(k_i^j - \theta^j\right)^2 \quad (57)$$

This function is also used, for example, in Baeriswyl and Cornand (2014) and Myatt and Wallace (2011).

In this economy $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{k\theta} = 2(1-r)$, $U_{K\theta} = U_{\sigma\sigma} = 0$. This implies that $\kappa = \kappa^* = 1$, $\alpha = \alpha^* = r > 0$ and $\rho = 0$. Thus, this economy is efficient and is characterized by strategic complementarity. The findings about social, regional and inter-regional value of information in this economy are as follows:

Corollary 5. *In efficient competitive economy with strategic complementarity described here,*

1. *the social and regional value of public and private information is positive, while inter-regional value of private information is negative;*

2. *the inter-regional value of public information about cost shock in region j may be negative if $n_j > 1/2$ and $r < \frac{2n-1}{n^2}$.*

As we can see, both regional and social value of information is positive. This means that social welfare is increasing in the precisions of both private and public information and that the information policy of a local authority would be socially efficient. Nevertheless, the inter-regional value of private information is negative. Private information is available only to the agents in the home region. This informational

[2] Note that private actions under complete information are given by $\kappa^j(\Theta) = \frac{A}{1+b} + (1 + b(1-n_j))\,\theta^j - b(1-n_j)\theta^{-j}$. The term $\frac{A}{1+b}$ in this expression arises because $U_k(0,0,0,0) = \frac{A}{1+b} \neq 0$. This does not change the results.

asymmetry prevents the foreign agents from the efficient inter-region coordination. An increase in the precision of private information increases this asymmetry and creates a negative inter-regional externality. Moreover, public information about the larger region also creates a negative inter-regional effect, if the extent of strategic complementarity is relatively low.

6.2 Inefficient degree of coordination

Hellwig and Veldkamp (2009) study the model of price-setters with the following utility function:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = -\left(k_i^j - rK - (1-r)\,\theta^j\right),^2 \tag{58}$$

where k_i^j is a price of agent i, K is the average price in the economy, θ^j is a shock to the optimal price level. In this economy $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r^2$, $U_{k\theta} = 2\,(1-r)$, $U_{K\theta} = -2\,(1-r)$, $U_{\sigma\sigma} = 0$. This implies that $\kappa = \kappa^* = 1$, equilibrium degree of coordination is equal to $\alpha = r$ and efficient degree of coordination is equal to $\alpha^* = r\,(2-r)$. If $r > 0$, this model is characterized by strategic complementarity and efficient degree of coordination exceeds the equilibrium degree, meaning that $\alpha^* > \alpha > 0$. If $r < 0$, this model is characterized by strategic substitutability and efficient degree of coordination is lower than the equilibrium degree, meaning that $\alpha^* < \alpha < 0$.

The welfare properties of information are as follows:

Corollary 6. *In the economy described here,*

1. *if $r > 0$,*
   - (a) *social, regional and inter-regional value of public information is positive, while inter-regional value of private information is negative;*
   - (b) *regional and social values of private information in region j may be negative, if $rn_j > 1/2$;*
2. *if $r < 0$, there exists $\overline{r} < 0$ such that*
   - (a) *the social value of private information is positive if $r > \overline{r}$ and may be negative, if $r < \overline{r}$;*
   - (b) *social value of public information may be negative if $rn_j < -1$;*
   - (c) *the regional value of public and private information, inter-regional value of public information is positive, while inter-regional value of private information is negative.*

Part 1 of Corollary 6 indicates the role of information in the economy with strategic complementarity. The social value of public information is positive, which coincides with a one-region version of the model, as $\alpha^* > \alpha > 0$ suffices for that. The segmentation of the economy enlarges the set of value r, for which an increase in the precision of private information may be socially undesirable. In a one-region economy the social value of private information may be negative, if $r > 1/2$. In a two-region economy it may be negative, if $rn_j > 1/2$. In a two-region economy an increase in the precision of private information may be undesirable not only because it prevents agents inside the region from coordination, but also it disturbs the coordination between regions. Thus, in two-region economy private information is more likely to have a negative social value.

Part 2 of the Corollary 6 shows the value of information in the economy with strategic substitutability. In a one-region version of this economy, the social value of private information is necessarily positive, $\alpha^* < \alpha < 0$. In a two-region economy this is true only if the extent of substitutability is relatively low. This finding confirms the result listed in Part 2 of Proposition 4. In this economy the gap between the equilibrium and efficient degree of coordination is equal to $\alpha - \alpha^* = -r(1-r)$. This value depends negatively on r. Larger strategic substitutability means lower value of r and larger gap $\alpha - \alpha^*$. As we have seen in Proposition 4, the social value of private information is necessarily positive only if the gap between the equilibrium and efficient degree of coordination is not too large. Moreover, the social value of public information may also be negative for sufficiently large gap between the equilibrium and efficient degree of coordination.

6.3 Externality created by the dispersion in private actions

In a beauty-contest economy, described in Morris and Shin (2002), loss of a private agent is given by:

$$l_i = (1-r)(k-\theta_j)^2 + r(L_i - \bar{L}), \tag{59}$$

where $L_i = \int_0^1 (k_g - k_i)^2 \, dg$ represents the average distance between the action of the agent and the actions of all other private agents; $\bar{L} = \int_0^1 L_g \, dg$. This loss function is equivalent to the following utility function:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = -(1-r)\left(k_i^j - \theta^j\right)^2 - r\left(k_i^j - K\right)^2 + r\sigma_k^2$$

Thus, $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{\sigma\sigma} = 2r$, $U_{K\theta} = 0$, $U_{k\theta} = 2(1-r)$, $W_{KK} = -2(1-r)$, $W_{\sigma\sigma} = -2(1-r)$, $\rho = -\frac{r}{(1-r)} < 0$. This means that $\kappa = \kappa^* = 1$ and $\alpha = r > \alpha^* = 0$. This economy is characterized by two sources of inefficiency. First of all, the equilibrium degree of coordination is too large in comparison with the efficient degree of coordination. Secondly, the dispersion creates the positive externality for private agent, meaning that $U_{\sigma\sigma} > 0$ and $\rho < 0$. This source of inefficiency leads to a distortion in private actions under complete information. The equilibrium weight of the home fundamental in private actions is equal to $\kappa_{j,j} = 1 - r(1-n_j)$, while the efficient weight of the home fundamental is equal to $\kappa^*_{j,j} = 1$. Thus, the equilibrium weight of the home fundamental is lower than its efficient weight. The equilibrium weight of the foreign fundamental under complete information is equal to $\kappa_{j,-j} = r(1-n_j)$, while its efficient weight is equal to $\kappa^*_{j,-j} = 0$. Thus, the equilibrium weight of the foreign fundamental is higher than its efficient weight. Moreover, the socially optimal degree of coordination inside the region is equal to $\alpha^*_{j,j} = (1-n_j)\frac{r}{1-r}$, while the socially optimal degree of coordination between the regions is equal to $\alpha^*_{j,j} = -(1-n_j)\frac{r}{1-r}$. This means that some positive coordination inside the regions and the negative degree of coordination between regions are socially desirable. To understand this fact, let us remind that the private value of dispersion and the social value of the gap between regions is positive, as $U_{\sigma\sigma} > 0$. Thus, the negative coordination between regions and increased coordination inside them lead to a higher gap, which is socially desirable. This is different from a one-region version of the model, for which the efficient extent of coordination is equal to zero.

It is easy to show that in one-region version the social loss is decreasing in the precision of public information, if $r < 1/2$, and may be decreasing in the precision of public information, if $r > 1/2$. This means that the social value of public information may be negative only if the extent of strategic complementarity is relatively high. The social value of private information is always positive in a one-region version. The welfare properties of information in a two-region version of the model are listed in the following Corollary:

Corollary 7. *For the two-region beauty contest model of Morris and Shin (2002),*

1. *the social and the regional value of private information is positive for any (r,n);*
2. *for any n, there exists $\overline{r}_S \in (0,1)$ such that the social value of public information is positive if $r \geq \overline{r}_S(n)$ and may be negative, if $r < \overline{r}_S(n)$. Moreover, $\overline{r}_S(n) \geq 1/2$ and $\frac{\partial \overline{r}_S(n)}{\partial n} > 0$;*

3. *if $n \leq 1/2$, the regional value of public information is positive. If $n > 1/2$, there exists $\bar{r}_j \in (0,1)$ such that the regional value of public information is positive if $r \geq \bar{r}_j(n)$ and may be negative, if $r < \bar{r}_j(n)$;*

4. *the inter-regional value of private information may be negative, if $n_j > \frac{1}{4-r}$; the inter-regional value of public information may be positive, if $n_j < \frac{1-\alpha}{2-\alpha}$.*

Part 1 of Corollary 7 indicates that the social value of private information in a two-region economy is positive, as it is in a one-region model. Part 2 shows that the social value of public information may be of negative social value, if r is sufficiently small. This contradicts to a one-region model, when the social value of public information may be negative for relatively high values of r. This distinction comes from the fact, that in two-region version there are two sources of inefficiency. An increase in r means not only an increase in the equilibrium degree of coordination, but also an increase in discrepancy created by the cross-sectional dispersion. As $\frac{\partial(\alpha^*-\alpha)}{\partial r} < 0$ and $\frac{\partial \underline{\rho}}{\partial(\alpha^*-\alpha)} > 0$, an increase in r makes it more likely that the externality becomes too low to get the negative social value of public information. On the contrary, a decrease in r leads to an increase in threshold $\underline{\rho}$ in Proposition 5, making the social value of information negative. Part 3 of Corollary 7 shows, that the regional value of public information is positive, if $n \leq 1/2$. This implies that the authority in a small region would overestimate the value of public information. Thus, the informational policy of local authorities in this economy may be too transparent from the social point of view.

7 Conclusion

In this paper we study social, regional and inter-regional value of information in segmented economies. We show that the externalities, which arise due to strategic and informational spillovers between regions, change considerably the welfare properties of information. For example, in economies which are efficient in a one-region model, the social value of public information may be negative in a two-region model, if the agents value dispersion in private actions.

This finding gives rise to two concerns about information policy elaboration. The first concern is about using representative-agent models. We show that the policy, elaborated on a base of such models, may be inefficient if the economy is segmented in reality. The second concern is about potential inefficiency of information policies, if they are elaborated by the local authorities. As the regional and the social values of information can differ, the regional authority may choose the policy, which is either too transparent or too opaque from the social perspective. We apply this methodology to several examples, which illustrate these issues.

The methodology can be further developed. First of all, in the current version we assume that strategic effects have global character. Distinction between local and inter-regional strategic effects may be an interesting extension of the model. Moreover, the fundamental shocks in our model are uncorrelated. Nevertheless, in reality all the economies are interconnected. The study of technological spillovers between regions would be another extension of the model.

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Appendix

A Proof of Proposition 3

If the equilibrium is globally efficient for any information structure, it should be efficient under complete information, implying that $\kappa_{j,k} = \kappa^*_{j,k}$ with $j,k \in \{1,2\}$. Moreover, the coordination degrees in equilibrium with incomplete information should coincide with the coordination degrees in the globally optimal distribution, implying that $\alpha_{j,k} = \alpha^*_{j,k}$ with $j,k \in \{1,2\}$.

The gap between the equilibrium and the globally efficient degrees of regional coordination is as follows:

$$\alpha_{j,j} - \alpha^*_{j,j} = (\alpha - \alpha^*)\, n_j - \rho\,(1 - n_j) \tag{60}$$

The gap between the equilibrium and the globally efficient degrees of inter-regional coordination is as follows:

$$\alpha_{j,j} - \alpha^*_{j,j} = (\alpha - \alpha^*)\,(1 - n_j) + \rho\,(1 - n_j) \tag{61}$$

From (60) and (61), it is obvious, that both gaps are equal to zero if and only if $\alpha = \alpha^*$and $\rho = 0$. Analogically, the gap between the equilibrium and the globally efficient local distribution is as follows:

$$\kappa_{j,j} - \kappa^*_{j,j} = (\kappa - \kappa^*)\, n_j + \kappa\rho\,(1 - n_j)\,(1 - \alpha) \tag{62}$$

The gap between the equilibrium and the globally efficient inter-regional distribution is as follows:

$$\kappa_{j,j} - \kappa^*_{j,j} = (\kappa - \kappa^*) n_j + \kappa\rho (1 - n_j)(1 - \alpha) \tag{63}$$

For both (62) and (63) to be equal to zero, two conditions must held: $\kappa = \kappa^*$and $\rho = 0$. From these two findings, Proposition 3 comes immediately.

B Proof of Proposition 4

Part 1 The global social loss in efficient economies with $\alpha = \alpha^*$, $\kappa = \kappa^*$ and $\rho = 0$ is equal to the following:

$$\mathbf{L}_{s,1} = \frac{n_j(1-\alpha)\left(\sigma_z^{-2}(1-\alpha(1-n_j)) - \sigma_x^{-2}\alpha^2 n(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2}+(1-\alpha n_j)\sigma_x^{-2}\right)} (64)$$

The function (64) is decreasing in both precisions: $\frac{\partial L_{s,1}}{\partial \sigma_z^{-2}} < 0$ and $\frac{\partial L_{s,1}}{\partial \sigma_x^{-2}} < 0$. Part 1 of Proposition 4 comes immediately.

Part 2 The global social loss in economies with inefficient degree of coordination ($\alpha \neq \alpha^*$, $\kappa = \kappa^*$ and $\rho = 0$) is as follows:

$$L_{s,2} = L_{s,1} + n_j^2 (\alpha^* - \alpha) \hat{L}_{s,2} \tag{65}$$

$$\hat{L}_{s,2} = \frac{\left(\sigma_z^{-2} + \sigma_x^{-2}\alpha(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2} + (1-\alpha n_j)\sigma_x^{-2}\right)^2}$$

Term $\hat{L}_{s,2}$ is decreasing in the precision of public information, if $\alpha > 0$ and my be increasing, if $\alpha < 0$. Thus, condition $\alpha^* > \alpha > 0$ suffices for the positive value of public information. The derivative of this term over the precision of private information:

$$\frac{\partial \hat{L}_{s,2}}{\partial \sigma_x^{-2}} = \frac{2\left(\sigma_z^{-2} + \sigma_x^{-2}\alpha(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2} + (1-\alpha n_j)\sigma_x^{-2}\right)^3} \tag{66}$$

The numerator is negative if $\alpha < \frac{\sigma_z^{-2}}{\sigma_x^{-2}(1-n)}$, thus condition $\alpha^* < \alpha < 0$ is not sufficient for the global value of private information to be positive. Taking derivative of (65) over the precision of private information in this case, we get that the loss is decreasing over the precision if $\alpha^* > \alpha - \frac{(1-\alpha)(1-\alpha n)}{2\alpha n(1-n)}$. This gives Part 2 of the proposition.

Part 2 The global social loss in inefficient economies ($\alpha \neq \alpha^*$, $\kappa \neq \kappa^*$ and $\rho = 0$) is as follows:

$$L_{s,3} = L_{s,2} + 2n_j^2 \hat{L}_{s,3} \frac{\kappa^* - \kappa}{\kappa} \tag{67}$$

$$\hat{L}_{s,3} = \frac{(1-\alpha^*)\left(\sigma_z^{-2} + \sigma_x^{-2}\alpha(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2} + (1-\alpha n_j)\sigma_x^{-2}\right)} \tag{68}$$

Term $\hat{L}_{s,3}$ is decreasing in the precision of private information:

$$\frac{\partial \hat{L}_{s,3}}{\partial \sigma_x^{-2}} = -\frac{(1-\alpha^*)(1-\alpha)}{\sigma_z^{-2}\left(\sigma_z^{-2} + (1-\alpha n_j)\sigma_x^{-2}\right)^2} < 0$$

Thus, for any strategic effect, sufficiently high level of gap $\frac{\kappa^*-\kappa}{\kappa}$ guarantees the positive value of private information.

The derivative of term $\hat{L}_{s,3}$ over the precision of public information:

$$\frac{\partial \hat{L}_{s,3}}{\partial \sigma_z^{-2}} = -\frac{(1-\alpha^*)\left(\zeta^2 + 2\alpha(1-n)\zeta + \alpha(1-n)(1-\alpha n)\right)}{\left(\sigma_z^{-2}\right)^2\left(\sigma_z^{-2} + (1-\alpha n_j)\sigma_x^{-2}\right)^2}, \tag{69}$$

where $\zeta = \frac{\sigma_z^{-2}}{\sigma_x^{-2}}$. As we can see, expression (69) is negative, if $\alpha > 0$, and may be positive, if $\alpha < 0$. The Part 3 of Proposition 4 comes immediately.

C Proof of Proposition 5

Part 1 The effect of externality on the marginal loss of the public information is given by the following derivative:

$$\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial \rho} = \frac{2n(1-\alpha)^2(1-n)\left(\zeta^2 - 2\alpha n\zeta - \alpha n(1-\alpha n)\right)}{(\zeta)^2(\zeta+1-\alpha n)^2\left(\sigma_x^{-2}\right)^2} \tag{70}$$

In case of strategic complementarity, expression 70 is negative for small values of the relative precision of public information ζ. It means that a decrease in ρ to a sufficiently large negative values would lead to a negative social value of public information. In case of strategic substitutability, expression 70 is positive for all values of the relative precision of public information ζ. It means that sufficiently high positive value of ρ is sufficient for the negative value of public information.

Part 2 The effect of externality on the marginal loss of the private information is given by the following derivative:

$$\frac{\partial^2 L_s}{\partial \sigma_x^{-2}\partial \rho} = \frac{2n(1-\alpha)^2(1-n)}{(\zeta)^2(\zeta+1-\alpha n)^2\left(\sigma_x^{-2}\right)^2} \tag{71}$$

Expression 71 is positive, meaning that sufficiently high value of ρ ensures the socially negative value of private information.

Part 3 The effect of the gap between efficient and equilibrium distributions under complete information of the marginal losses is as follows:

$$\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial(\kappa^* - \kappa)} = -\frac{2n^2(1-\alpha^*)\left(\zeta^2 + 2\alpha(1-n)\zeta + \alpha(1-n)(1-\alpha n)\right)}{(\zeta)^2(\zeta+1-\alpha n)^2\left(\sigma_x^{-2}\right)^2} \tag{72}$$

In case of strategic complementarity, this expression is negative. Using the implicit function theorem:

$$\frac{\partial \underline{\rho}}{\partial(\kappa^* - \kappa)} = -\frac{\left.\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial(\kappa^*-\kappa)}\right|_{\rho=\underline{\rho}}}{\left.\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial \rho}\right|_{\rho=\underline{\rho}}}, \tag{73}$$

where both numerator and denominator are negative, implying that $\frac{\partial \underline{\rho}}{\partial(\kappa^*-\kappa)} < 0$. Analogically, $\frac{\partial \overline{\rho}}{\partial(\kappa^*-\kappa)} < 0$.

D Proof of Proposition 6

The proof coincides with the proof of Propositions 4 and 5, applied for the regional loss component:

$$
\begin{aligned}
L_j = {} & n_j \frac{|W_{KK}|}{2} \left[Var\left(K-\bar{\kappa}\right) + 2Cov\left(K-\bar{\kappa};\bar{\kappa}-\bar{\kappa}^*\right) + 2Cov\left(K-\bar{\kappa};\bar{\kappa}^*-\tilde{\kappa}^j\right)\right] + \\
& + n_j\left(1-n_j\right)|U_{kk}|\left(1-\alpha\right)\left[Cov\left(K-\bar{\kappa};K_j-\kappa_j-\left(K_{-j}-\kappa_{-j}\right)\right) + Cov\left(K-\bar{\kappa};\kappa_j-\kappa_j^*-\left(\kappa_{-j}-\kappa_{-j}^*\right)\right)\right] \\
& + n_j\left(1-n_j\right)|U_{kk}|\left(1-\alpha\right)\left[Cov\left(K-\bar{\kappa};\kappa_j^*-\tilde{\kappa}_j^j-\left(\kappa_{-j}^*-\tilde{\kappa}_{-j}^j\right)\right)\right] + \\
& + n_j\left(1-n_j\right)|U_{kk}|\left(1-\alpha\right)\left[Cov\left(\bar{\kappa}-\bar{\kappa}^*;K_j-\kappa_j-\left(K_{-j}-\kappa_{-j}\right)\right) + Cov\left(\bar{\kappa}^*-\tilde{\kappa}^j;K_j-\kappa_j-\left(K_{-j}-\kappa_{-j}\right)\right)\right] \\
& + n\left(1-n\right)\frac{|U_{kk}+nU_{\sigma\sigma}|}{2}\left[Var\left(K_1-\kappa_1-\left(K_2-\kappa_2\right)\right) + 2Cov\left(K_1-\kappa_1-\left(K_2-\kappa_2\right);\kappa_1-\kappa_1^*-\left(\kappa_2-\kappa_2^*\right)\right)\right] \\
& + n\left(1-n\right)\frac{|U_{kk}+nU_{\sigma\sigma}|}{2}\left[2Cov\left(K_1-\kappa_1-\left(K_2-\kappa_2\right);\kappa_1^*-\tilde{\kappa}_1^1-\left(\kappa_2^*-\tilde{\kappa}_2^1\right)\right)\right] \\
& + n\left(1-n\right)\frac{|U_{kk}+nU_{\sigma\sigma}|}{2}\sigma_{k,j}^2 - n\left(1-n\right)\frac{U_{\sigma\sigma}}{2}\sigma_{k,-j}^2
\end{aligned}
\tag{74}
$$

Substituting here the corresponding variances from the main text with $\kappa = \kappa^*$, $\alpha = \alpha^*$ and taking the derivatives gives Propositions 6, 7 and 8.